General theory of storm surge
1D and 2D description

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Storm surge is an abnormal rise of water associated with a cyclone, not including tidal influences

Low pressure system can be a baroclinic cyclone, tropical cyclone, or a hybrid of the two.

Fundamental surge components

- Pressure setup - *increase in water level due to lower atmospheric pressure in storm interior.* A slight surface bulge occurs within the storm, greatest at the storm's center, decreasing at the storm's periphery. For every 10-mb pressure drop, water expands 4.0 inches.
  - *Effect is a constant*

- Wind setup - *increase in water level due to the force of the wind on the water.* As the transported water reaches shallow coastlines, bottom friction slows their motion, causing water to pile up. Further enhanced near land boundaries.
  - *Depends on bathymetry, size, and intensity. MOST IMPORTANT IN TERMS OF MAGNITUDE FOR SHALLOW WATER BATHYMETRIES!*

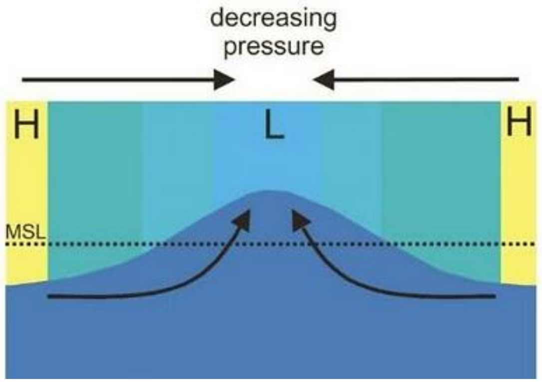
- Geostrophic adjustment – *water levels adjust to a developing longshore current.*

  - *Impact increases for slow-moving tropical cyclones*
  - *Impact increases for larger tropical cyclones*
  - *Causes a storm surge "forerunner"*
  - *Generally second in importance. Impact varies with bathymetry slope and intensity*

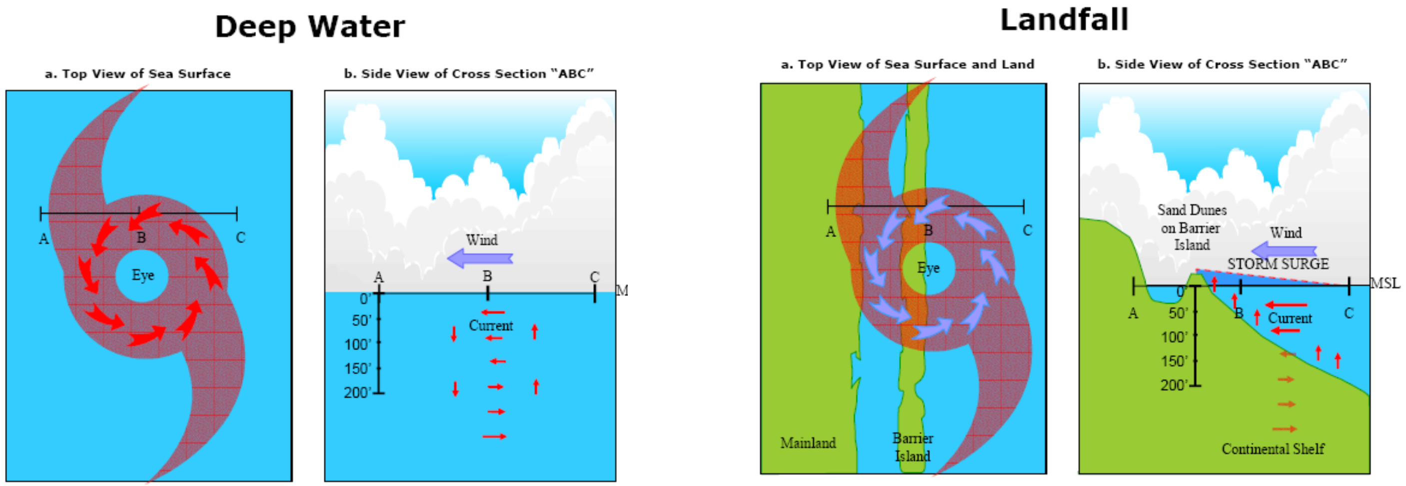
- Wave setup - *increase due to onshore waves.* Incoming water from wave breaking exceeds retreating water, resulting in water accumulation.

  – *Impact minor in shallow bathymetry; may be most important in deep bathymetry (still the subject of research)*

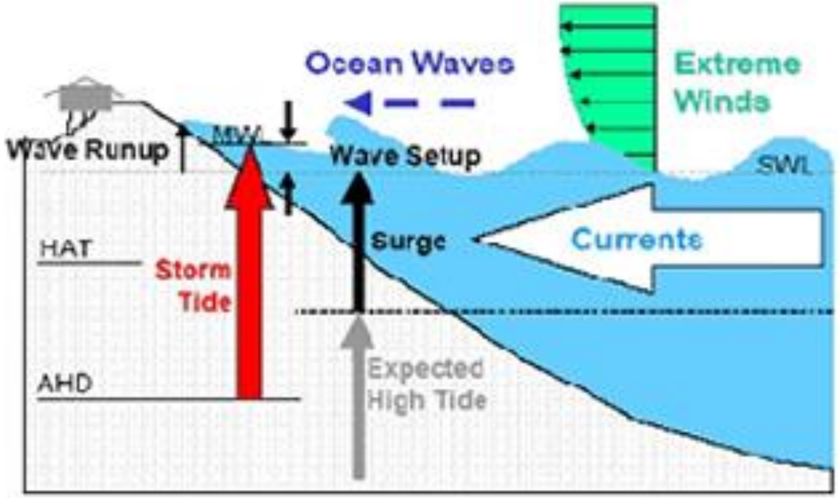
Pressure setup



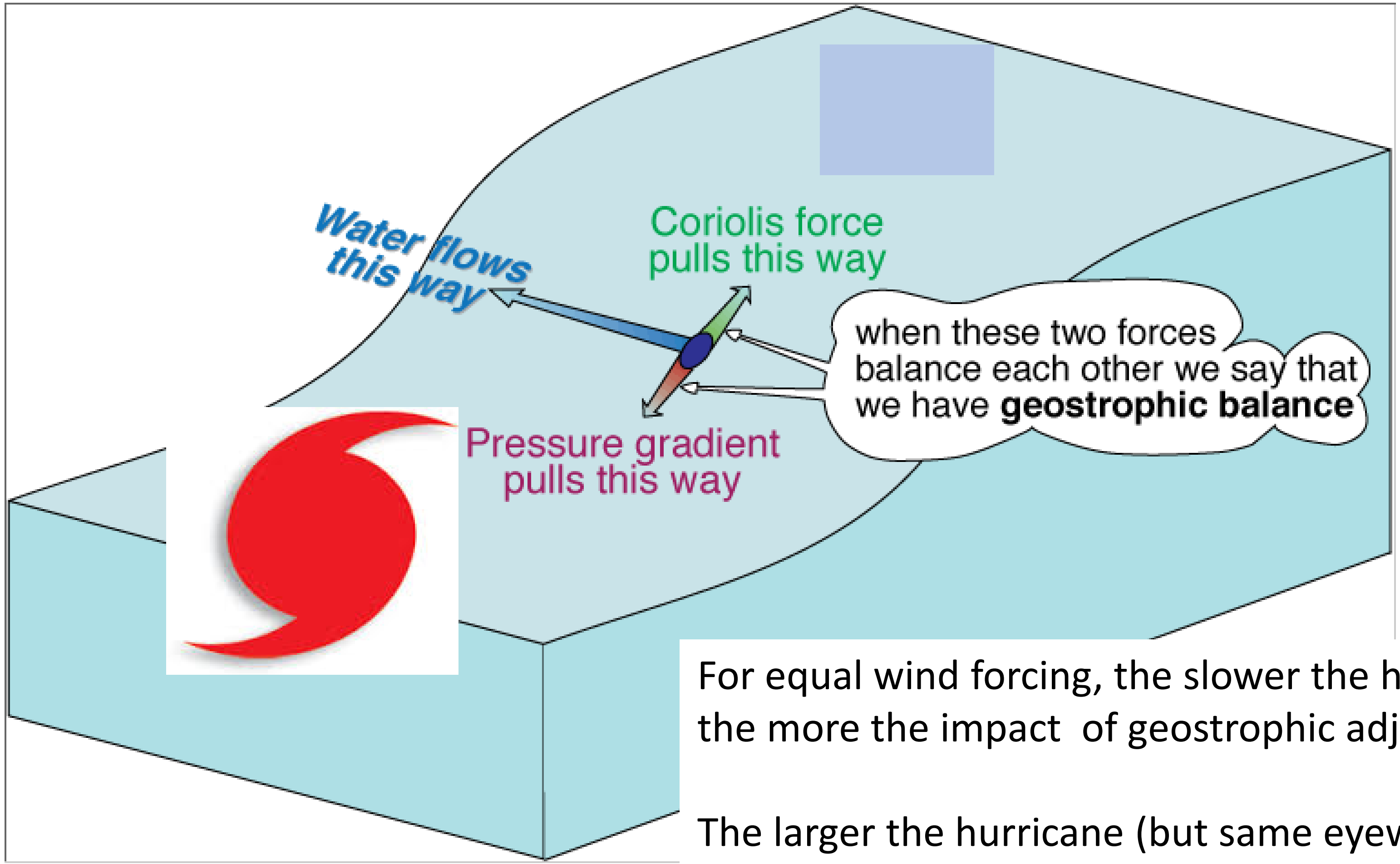
Wind setup



Wave setup



Geostrophic adjustment (creates surge "forerunner")



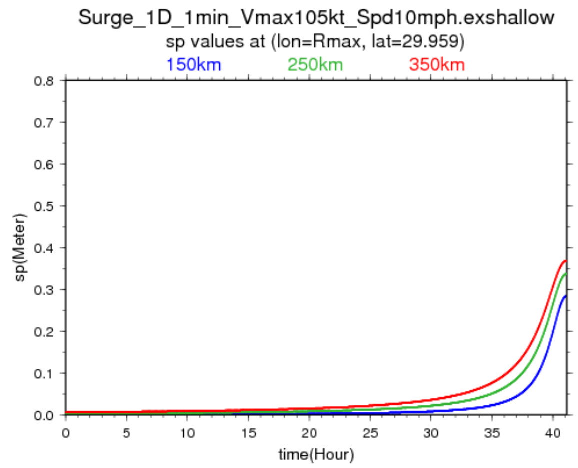
The balance between pressure gradient forces and Coriolis forces on a parcel

For equal wind forcing, the slower the hurricane moves, the more the impact of geostrophic adjustment on surge

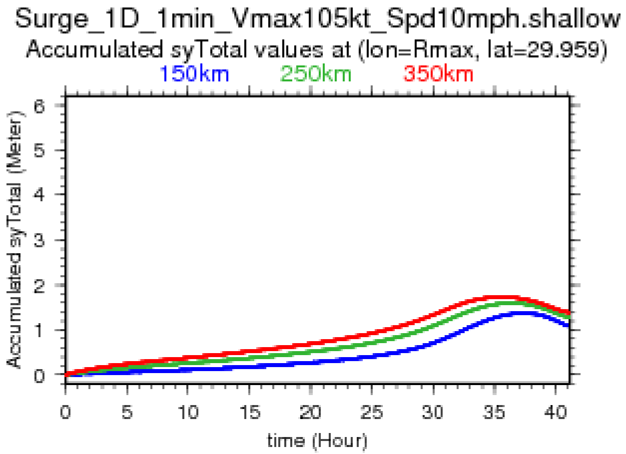
The larger the hurricane (but same eyewall wind magnitude), the more the impact of geostrophic adjustment on surge (Fitzpatrick et al. 2012)

http://www.seos-project.eu/modules/oceancurrents/oceancurrents-c06-s02-p01.html

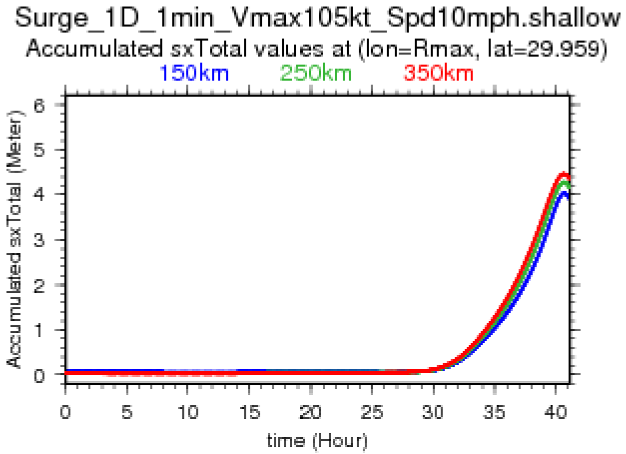
Pressure effect
(peaks at landfall)



Surge forerunner
(peaks before landfall)



Wind effect
(peaks at landfall)



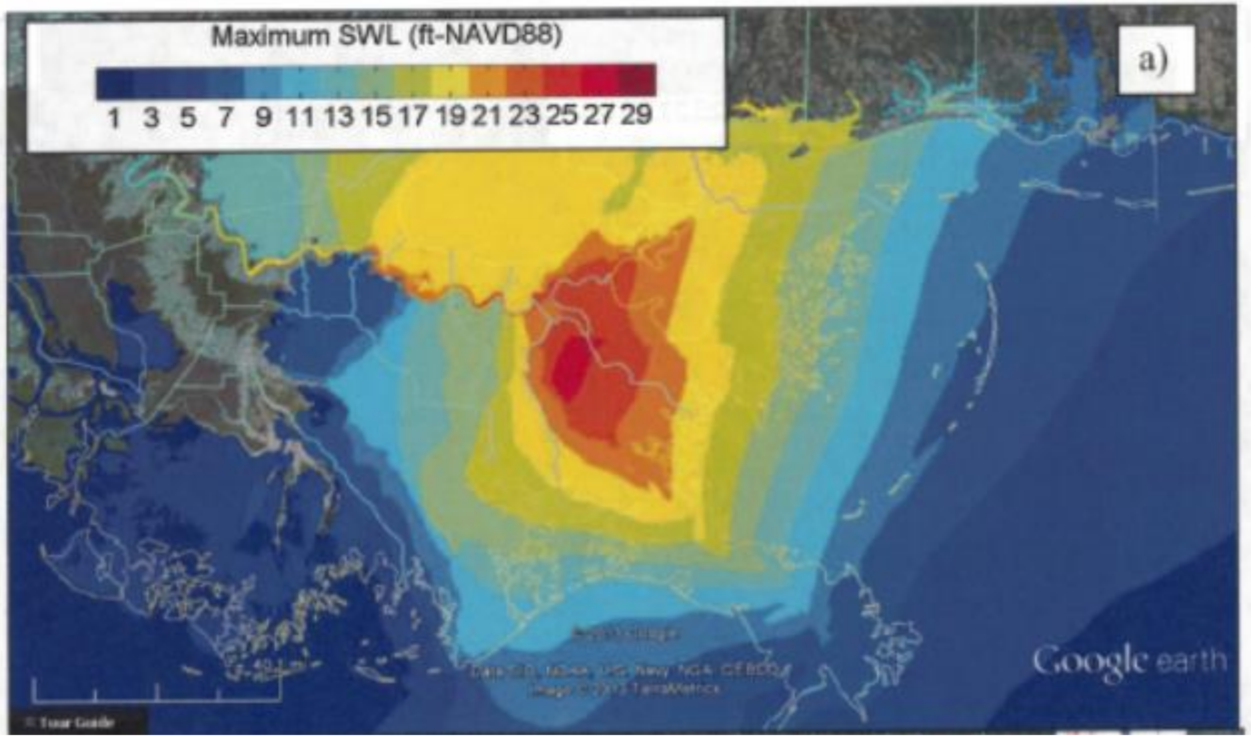
Time series example for Cat 3 in shallow bathymetry for small, average, and large hurricane moving 10 mph

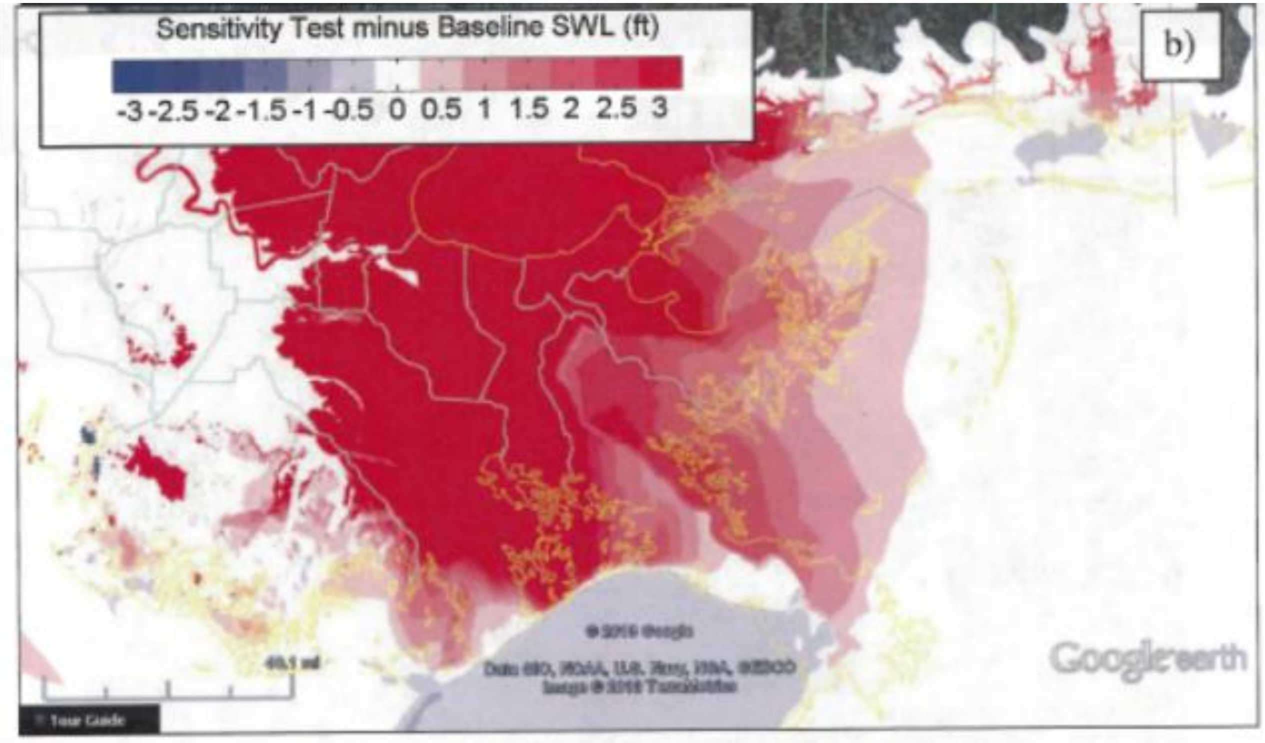
Surge on coastline

“Size” dictated by radius of 34 knots winds

Eyewall winds same magnitude

Comparison avg speed versus slow storm
Same spatial wind stress structure





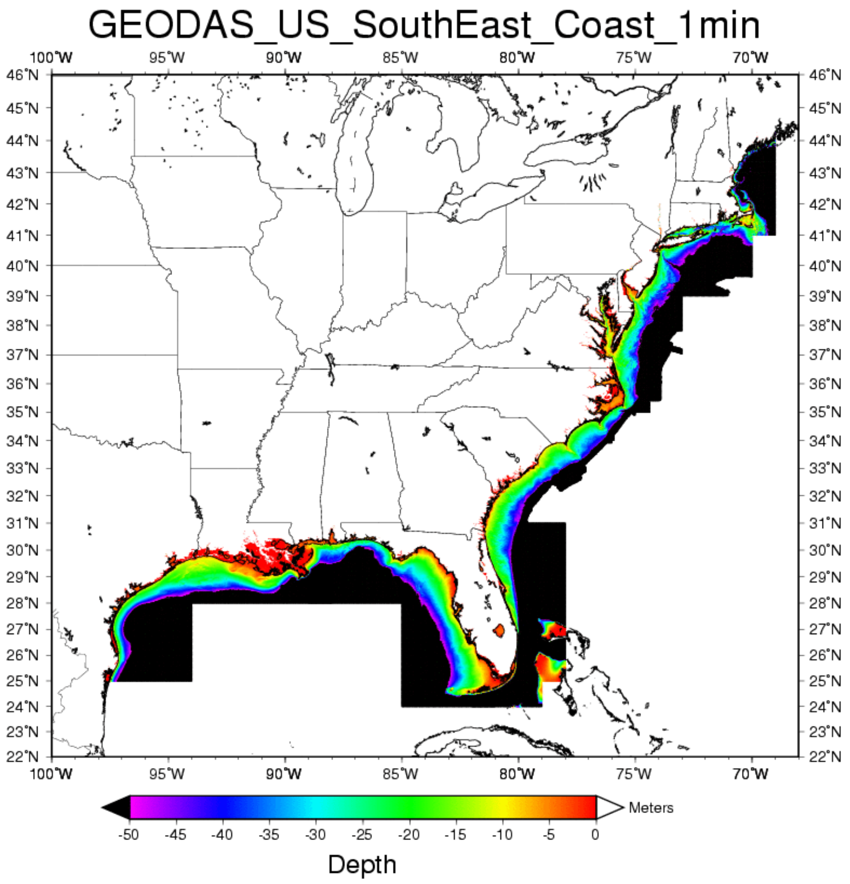
Other components for consideration

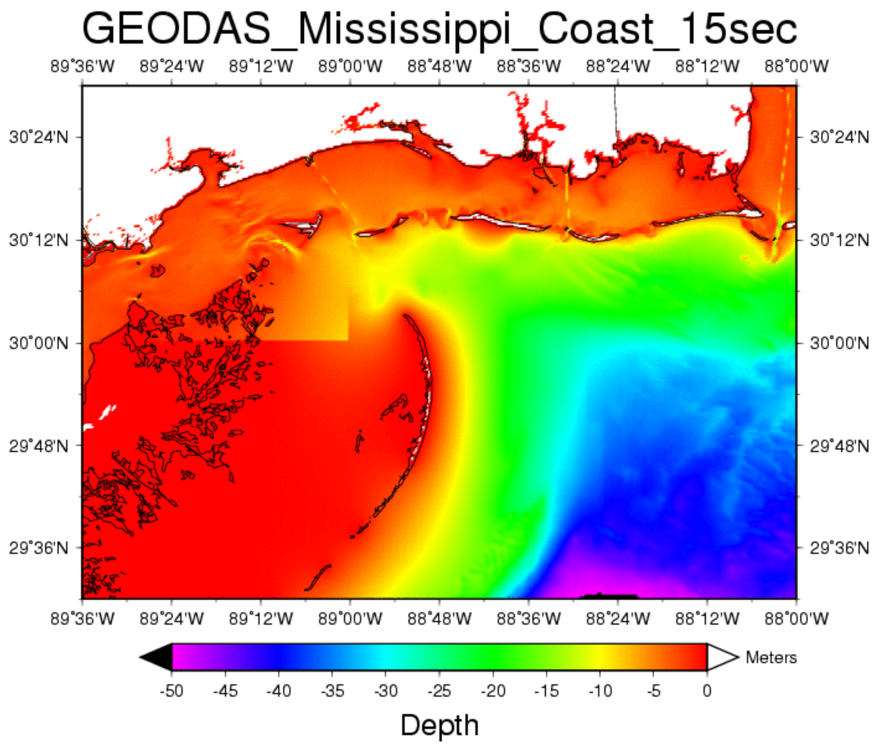
- Tide
- Steric setup – (water expansion or contraction as function of water temperature, small)
- Nonlinear advection (small, neglected in SLOSH, optional in ADCIRC)
- Dissipation terms

Note that, in two dimensions, all eight interactions become more complicated

ADCIRC has a river hydrology option as well

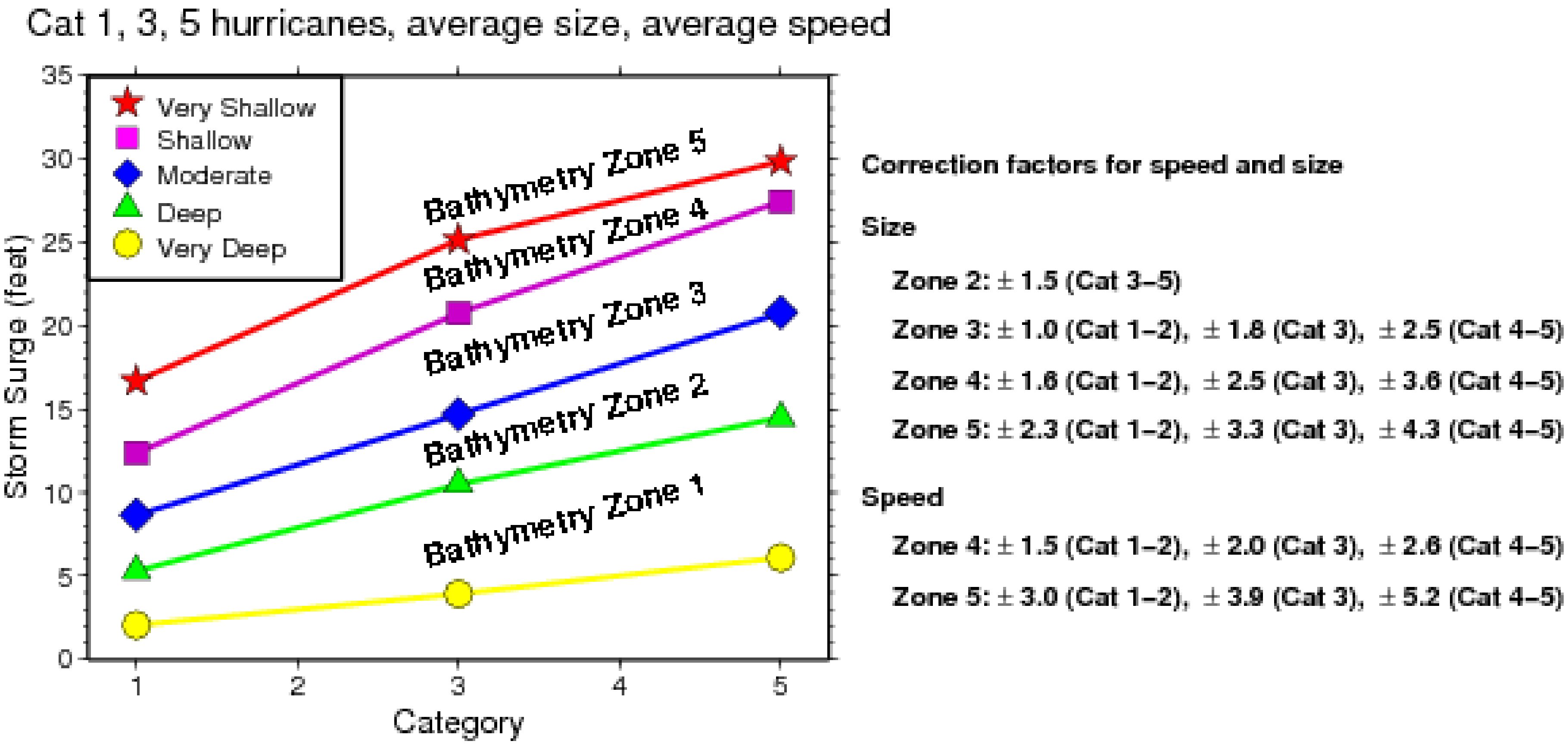
Surge varies due to different bathymetries and boundaries





Thought question --- where would surge be worse for a major hurricane?

Effect of hurricane intensity, size, and speed on storm surge



Scale validated against storm surge database, publication in process

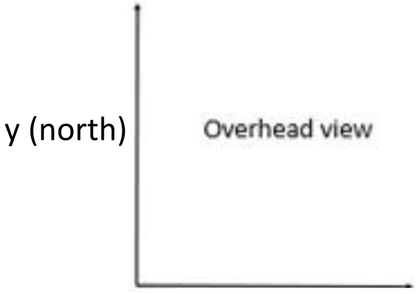
Equations in two dimensions

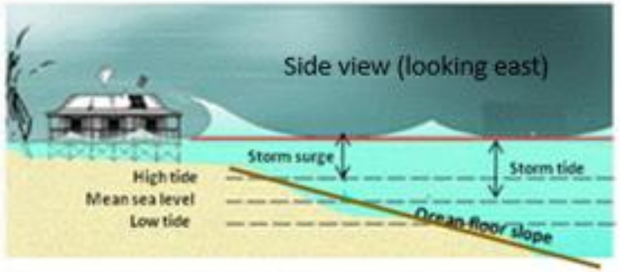
$$\frac{\partial u_{water}}{\partial t} + u_{water}\frac{\partial u_{water}}{\partial x} + v_{water}\frac{\partial u_{water}}{\partial y} = +f v_{water} - g\frac{\partial \eta}{\partial x} - \frac{1}{\rho_{water}}\frac{\partial p}{\partial x} + \frac{(\tau_{sx} - \tau_{bx})}{\rho_{water}(h+\eta)}$$

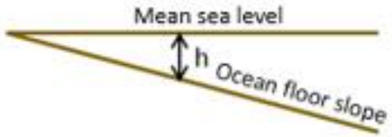
$$\frac{\partial v_{water}}{\partial t} + u_{water}\frac{\partial v_{water}}{\partial x} + v_{water}\frac{\partial v_{water}}{\partial y} = -f u_{water} - g\frac{\partial \eta}{\partial y} - \frac{1}{\rho_{water}}\frac{\partial p}{\partial y} + \frac{(\tau_{sy} - \tau_{by})}{\rho_{water}(h+\eta)}$$

$$\frac{1}{(h+\eta)}\frac{\partial \eta}{\partial t} = -\left(\frac{\partial u_{water}}{\partial x} + \frac{\partial v_{water}}{\partial y}\right)$$

Equations in one dimension







$$\frac{\partial u_{water}}{\partial t} + u_{water}\cancel{\frac{\partial u_{water}}{\partial x}} + \cancel{v_{water}\frac{\partial u_{water}}{\partial y}} = \cancel{+f v_{water}} - \cancel{g\frac{\partial \eta}{\partial x}} - \cancel{\frac{1}{\rho_{water}}\frac{\partial p}{\partial x}} + \frac{(\tau_{sx} - \tau_{bx})}{\rho_{water}(h+\eta)}$$

Assume small Assume small

$$\cancel{\frac{\partial v_{water}}{\partial t}} + u_{water}\cancel{\frac{\partial v_{water}}{\partial x}} + v_{water}\cancel{\frac{\partial v_{water}}{\partial y}} = -f u_{water} - g\frac{\partial \eta}{\partial y} - \frac{1}{\rho_{water}}\frac{\partial p}{\partial y} + \frac{(\tau_{sy} - \tau_{by})}{\rho_{water}(h+\eta)}$$

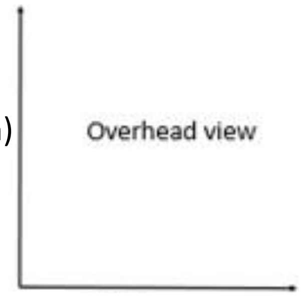
Assume small Assume small

$$\frac{1}{(h+\eta)}\frac{\partial \eta}{\partial t} = -\left(\cancel{\frac{\partial u_{water}}{\partial x}} + \cancel{\frac{\partial v_{water}}{\partial y}}\right)$$

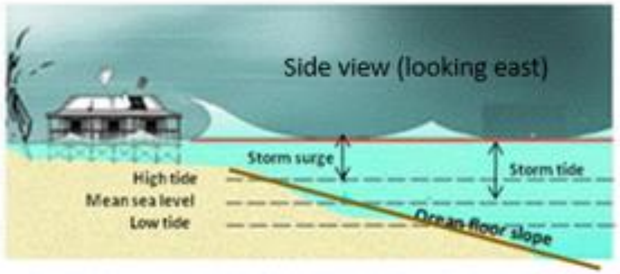
Assume small

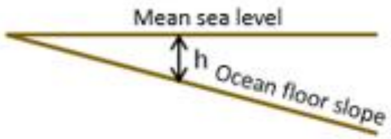
Basic storm surge 1D equations

y (north)



x (east)
assume parallel to beach





$$\frac{\partial u_{water}}{\partial t} = \frac{(\tau_{sx} - \tau_{bx})}{\rho_{water}(h+\eta)}; \ \tau_{sx} = -\rho C_D|\vec{V}|u; \ ; \ \tau_{bx} = -\rho_{water} C_k |\vec{V}_{water}| u_{water}$$

$$\frac{\partial \eta}{\partial y} = (wind\ setup) + (geostrophic\ adjustment) + (pressure\ setup) + (wave\ setup)$$

$$geostrophic\ adjustment = -\frac{f u_{water}}{g}$$

$$pressure\ setup = -\frac{1}{\rho_{water} g}\frac{\partial p}{\partial y}$$

$$wind\ setup = \frac{\tau_{sy}}{\rho_{water} g(h+\eta)}; \ \tau_{sy} = -\rho C_D |\vec{V}| v$$

Where:

f is Coriolis parameter, (2)(0.00007292)sin(latitude)

u_{water} is water current along coast; v_{water} is water current perpendicular to coast, assumed to be zero

P is air pressure

g is gravity

h is water depth

η is storm surge

ρ is density of air

ρ_{water} is density of water

u is zonal wind (west-east component); v is meridional wind (north-south component)

$|\vec{V}|$ is magnitude of wind vector; $|\vec{V}_{water}|$ is magnitude of ocean current vector

$$|\vec{V}| = \sqrt{u^2 + v^2}; \ |\vec{V}_{water}| = \sqrt{u_{water}^2 + v_{water}^2} \approx u_{water} \ \text{(since } v_{water} = 0\text{)}$$

C_k is ocean bottom drag coefficient

C_D is wind drag coefficient

τ_{sx} is zonal wind stress (west-east component); τ_{sy} is meridional wind stress (north-south component)

τ_{bx} is zonal bottom stress (west-east component)

Mathematics of wave setup

- Longuet-Higgins and Stewart (1962, 1964) showed that the increased water levels (the static "setup") results from the gradient of excess momentum flux associated with wave breaking (termed "radiation stress") in the surf zone balanced by the cross-shore pressure gradient (the slope of mean sea level).
- Assuming this balance and that breaking wave height is limited to a constant proportion γ of the total water, Bowen et al. (1968) derived

$$\frac{\partial \eta}{\partial y} = -\left(1 + \frac{8}{3\gamma^2}\right)^{-1} \frac{\partial h}{\partial y}$$

- After some scaling assumptions, Dean and Walton (2009) relate peak setup to breaking wave height H_B and γ

$$\eta_{max} = \frac{H_B}{\gamma + \frac{8}{3\gamma}}$$

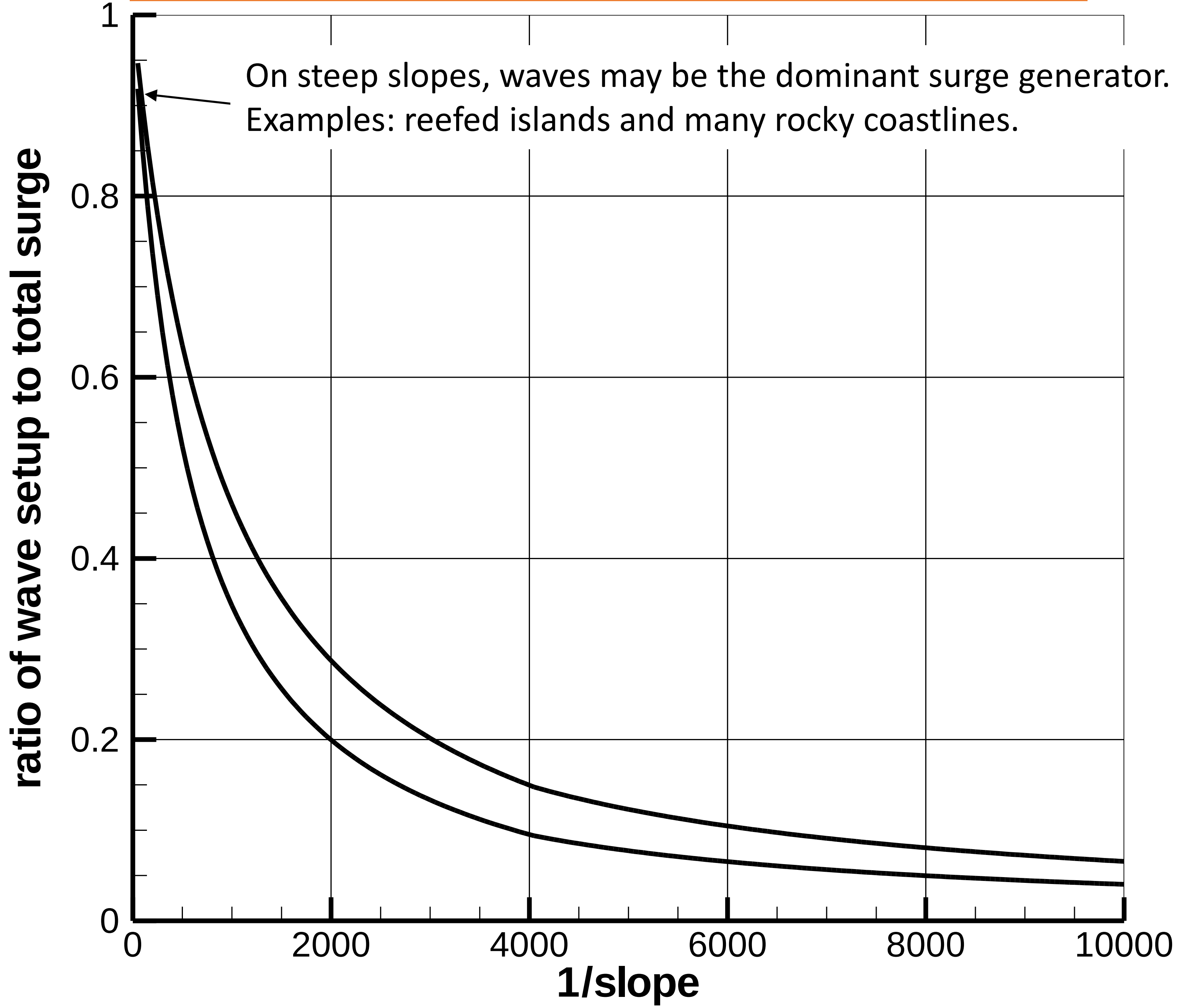
which, yields a limited range of ratios depending on assumed values for γ

$$A \leq \frac{\eta_{\max}}{H_B} \leq B \quad .$$

where A=10% and estimates for B vary from 20% to 40%

- Ultimately, wave setup is a function of $H_{B,}$ which hence depends on wave physics and bathymetry.
- Parametric schemes exist for wave setup, but not documented in this talk.

Estimated relative contribution of waves to total surge



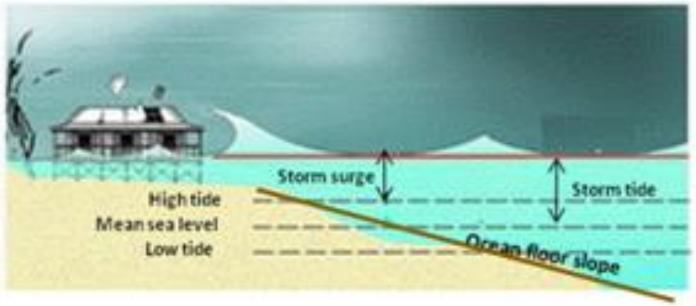
Resio (2012)

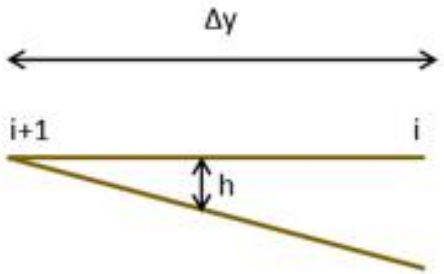
Examples of 1D surge model made available for class

- Python version, beta release
- FORTRAN version also available by request
- Shows example of forcing by hurricane parametric model
- Shows one methodology for modeling 1D surge equations

- Spreadsheet “toy surge model” also released

Spreadsheet exercise, 1D surge "toy model"





All units below are metric. Assume hurricane is making landfall so that the wind is perpendicular to shoreline: Then, $|\vec{V}| = \sqrt{u^2 + v^2}$ becomes v. For easier notation, assume this is the eyewall's maximum winds, typically denoted as V_{max}

At landfall, also assume no accelerations in the zonal current ($\frac{\partial u_{water}}{\partial t} = 0$)

Assume the ocean floor has a linear slope represented as

$$h_i = h_{i+1} + m(y_i - y_{i+1})$$

If the constants are C_D=3.5X10^{-3}, g=9.8ms^{-2}, ρ_{water}=1000 kg m^{-3} , and ρ=1 kg m^{-3} , the wind setup becomes

$$\frac{\partial \eta}{\partial y} = \frac{(3.57X10^{-7})V_{max}^2}{(h+\eta)}$$

Which can be solved from south to north, approximated as:

$$\eta_{i+1} = \eta_i + \frac{\Delta y(3.57X10^{-7})V_{max}^2}{(h_{i+1} + \eta_i)}$$

Where the last i is on the coastline (surge on land).

The pressure setup at landfall location is simply:

$$\eta_{land} = \frac{(p_{env} - p_c)}{\rho_{water}\, g}$$

Note this is approximately 1 cm per mb pressure drop (or 4.0 inches per 10 mb drop). Make sure pressure is in Pascals (1 mb=100 Pa).

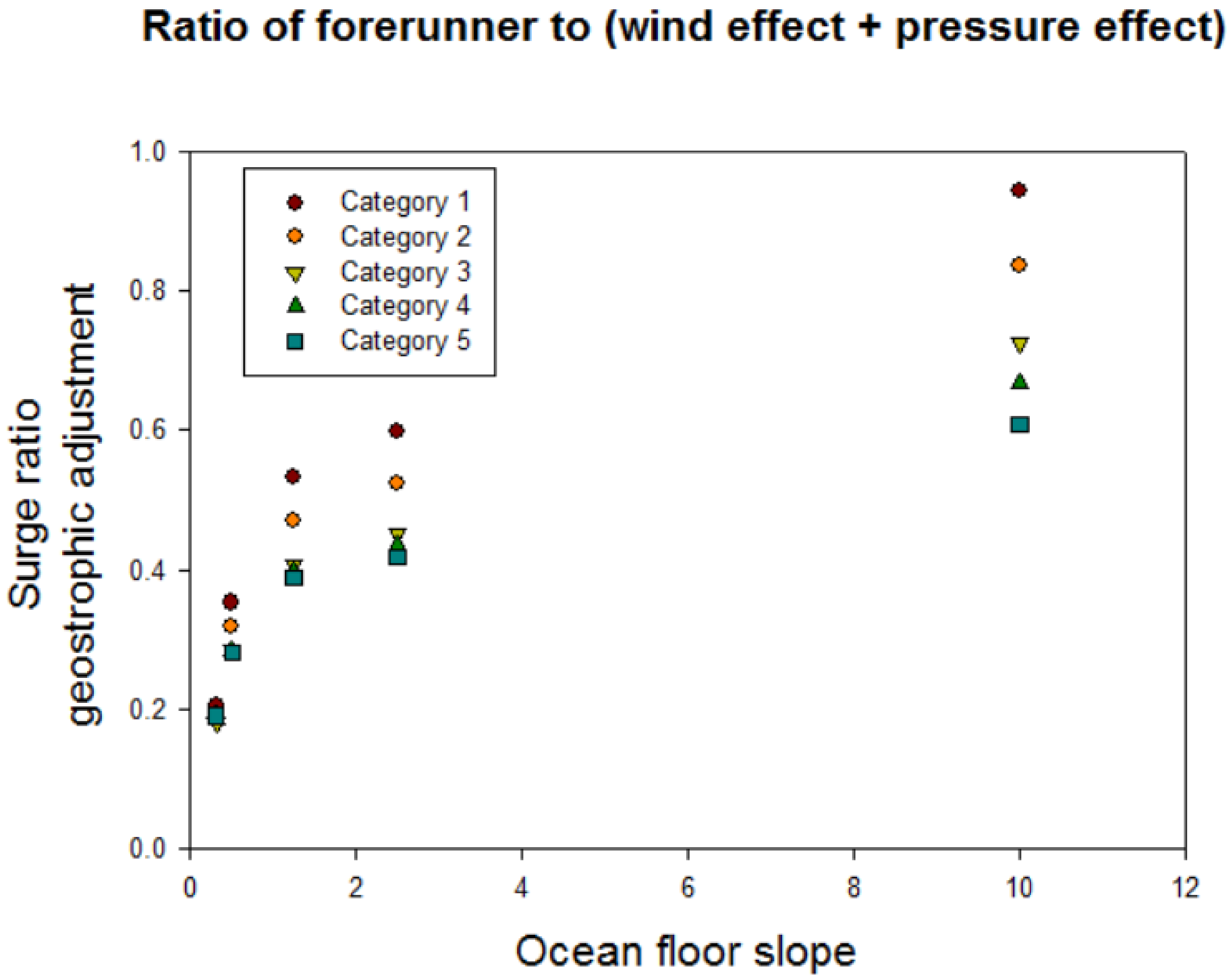
Central pressure can be roughly approximated by $p_c = p_{env} - 100(0.29V_{max})^{1.55}$ where $p_{env} = 101300$ Pa and V_{max} is in ms^{-1}.

In the spreadsheet vary the ocean slope m from 0.5 to 5.0 and wind speed V_{max} from 70 to 150 mph (cells D1 and G1).

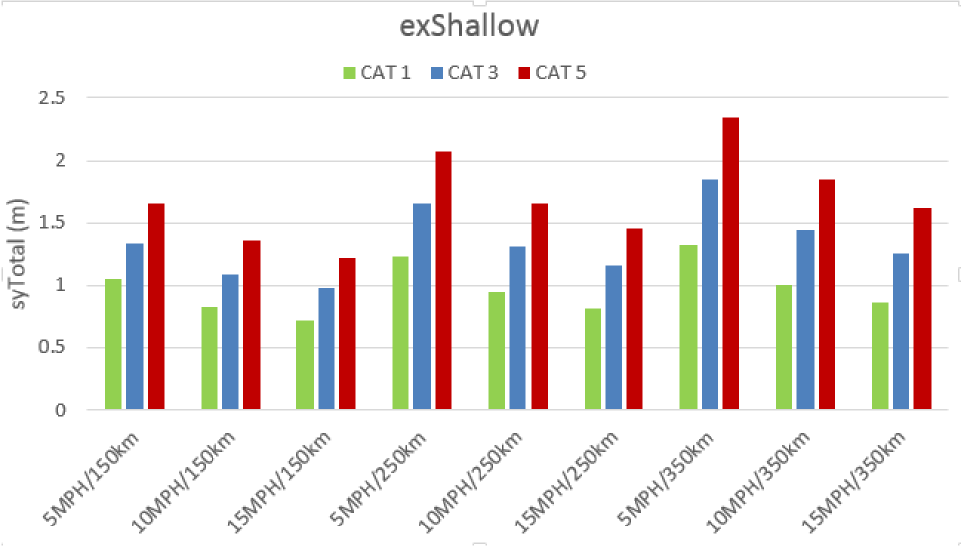
Parameterization for geostrophic adjustment

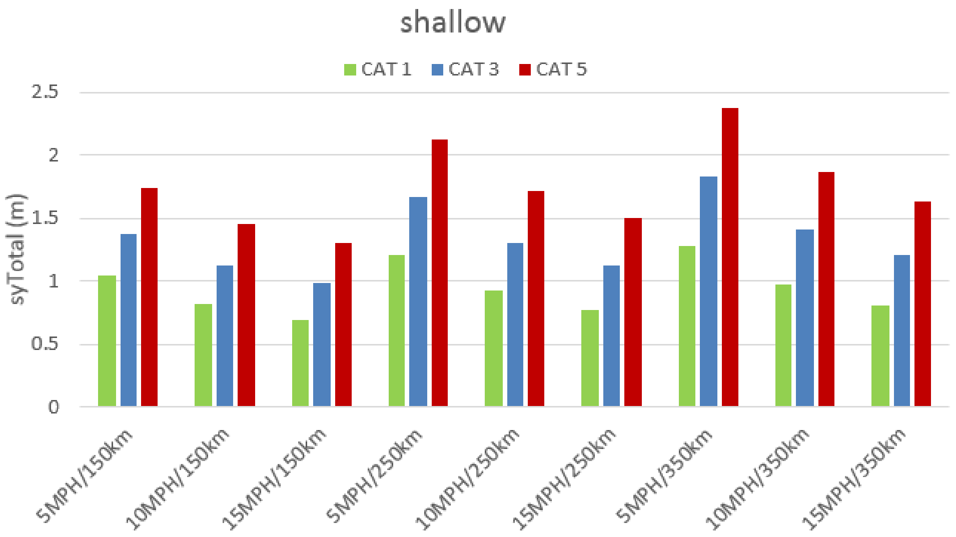
- Polynomial fit to slope, wind setup, and pressure setup

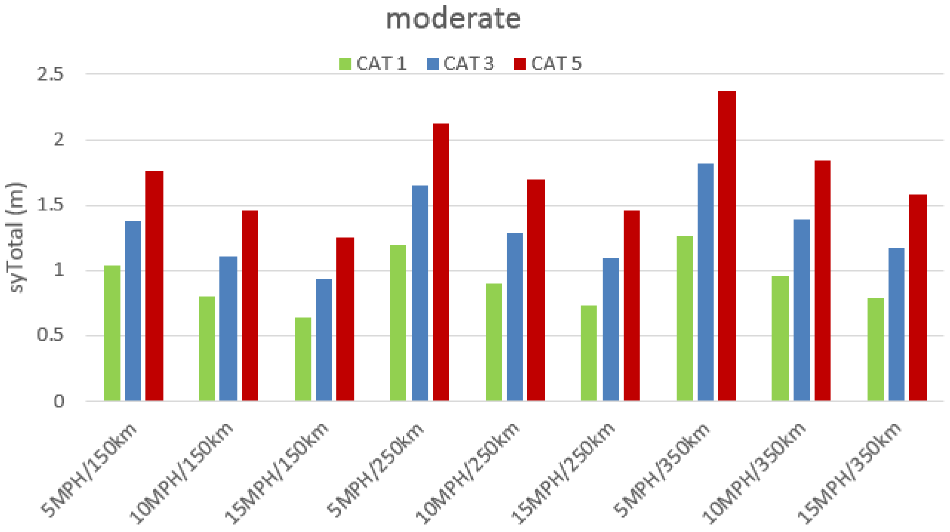
- Since geostrophic adjustment has a time component, there is variation due to storm size and storm translation speed.
- Impact proportional to size and translation speed
- Speed and size is ignored in the spreadsheet exercise. Fit is to avg size and avg speed

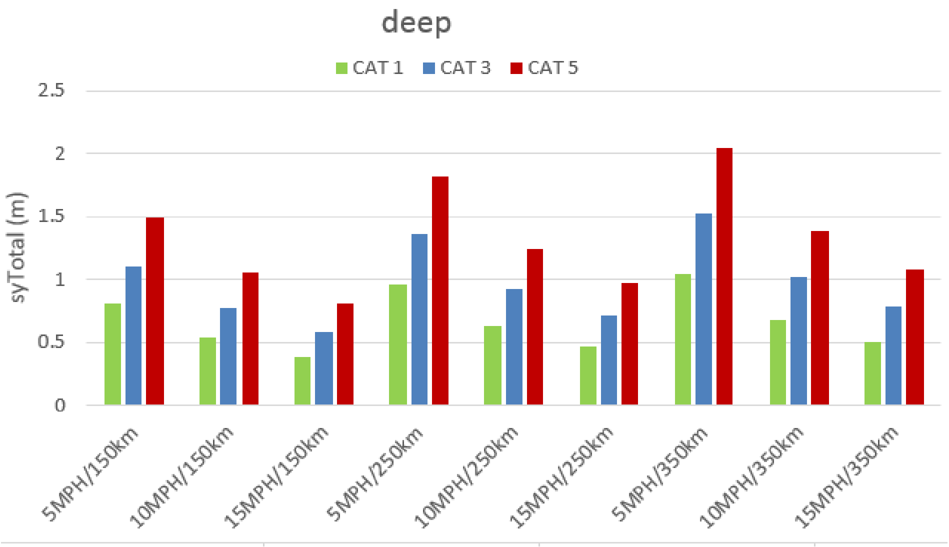


Geostrophic adjustment as a function of bathymetry, intensity, speed, and size









Role of longshore current

- Longshore current is a function of depth, wind, and wind duration (Bretschneider 1966):

$$u_{water} \sim |\overrightarrow{V}| tanh\left\{\frac{|\overrightarrow{V}|t}{(h+\eta)}\sqrt{\frac{1}{(h+\eta)^{1/3}}}\right\}$$

- Solution becomes asymptotic with time as geostrophic balance is approached…..but depth and wind change with time too, so role in storm surge generally needs to be modeled.

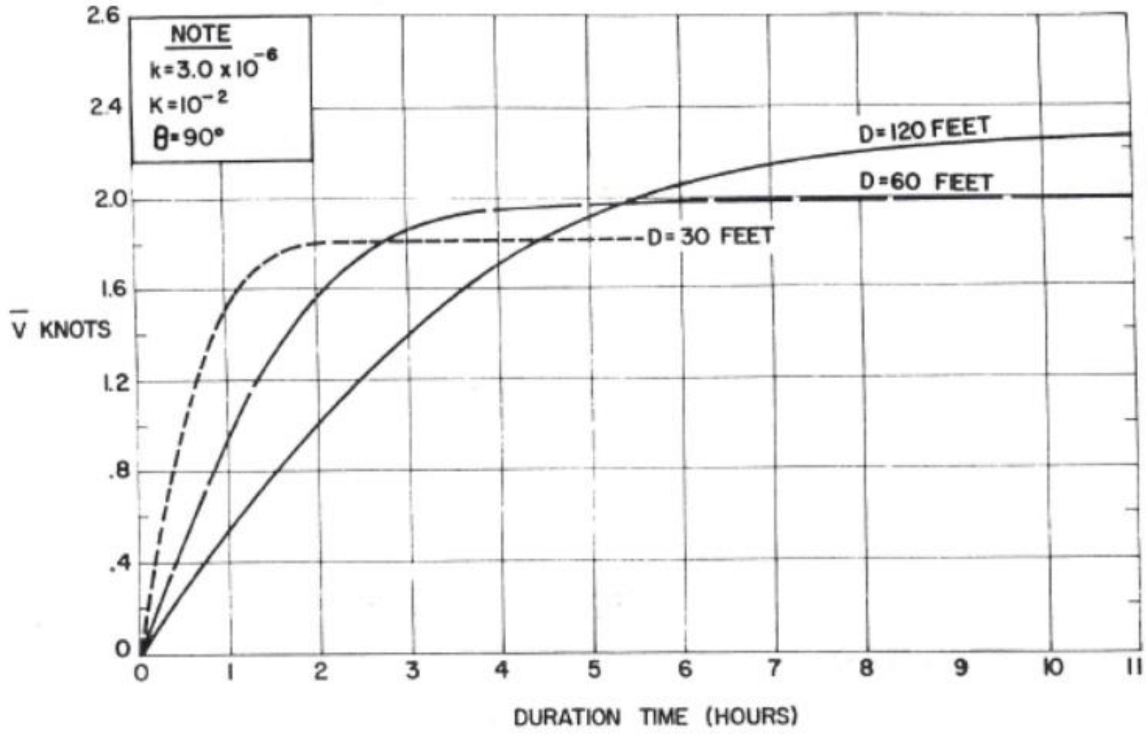


Figure 9
Generation of longshore wind currents for 60 knot winds

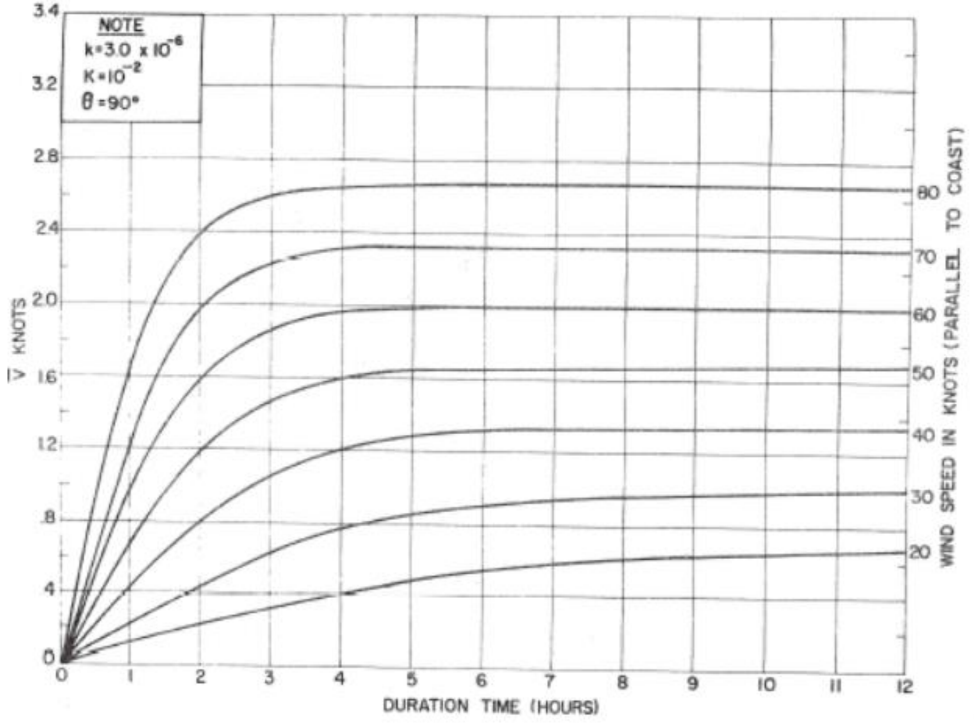


Figure 10
Generation of longshore wind currents for 60 foot water depth

Equations for ADCIRC and SLOSH

Recall equations in two dimensions

$$\frac{\partial u_{water}}{\partial t} + u_{water}\frac{\partial u_{water}}{\partial x} + v_{water}\frac{\partial u_{water}}{\partial y} = +f v_{water} - g\frac{\partial \eta}{\partial x} - \frac{1}{\rho_{water}}\frac{\partial p}{\partial x} + \frac{(\tau_{sx} - \tau_{bx})}{\rho_{water}(h+\eta)}$$

$$\frac{\partial v_{water}}{\partial t} + u_{water}\frac{\partial v_{water}}{\partial x} + v_{water}\frac{\partial v_{water}}{\partial y} = -f u_{water} - g\frac{\partial \eta}{\partial y} - \frac{1}{\rho_{water}}\frac{\partial p}{\partial y} + \frac{(\tau_{sy} - \tau_{by})}{\rho_{water}(h+\eta)}$$

$$\frac{1}{(h+\eta)}\frac{\partial \eta}{\partial t} = -\left(\frac{\partial u_{water}}{\partial x} + \frac{\partial v_{water}}{\partial y}\right)$$

SLOSH Equations

Notation "musical chairs"

SLOSH "h" same as class η
SLOSH "D" same as class h

Note SLOSH uses different notations for stress

Grid

Generally a polar, elliptical, or hyperbolic stretched finite difference curvilinear grid

$$\frac{\partial U}{\partial t} = -g(D+h)\left[B_r \frac{\partial (h-h_o)}{\partial x} - B_i \frac{\partial (h-h_o)}{\partial y}\right] + f(A_r V + A_i U) + C_r x_\tau - C_i y_\tau$$

$$\frac{\partial V}{\partial t} = -g(D+h)\left[B_r \frac{\partial (h-h_o)}{\partial y} + B_i \frac{\partial (h-h_o)}{\partial x}\right] + f(A_r U - A_i V) + C_r y_\tau + C_i x_\tau$$

$$\frac{\partial h}{\partial t} = -\frac{\partial U}{\partial x} - \frac{\partial V}{\partial y},$$

where U and V are the components of transport, g is the gravitational constant, D is the depth of quiescent water related to the National Geodetic Vertical Datum (NGVD 29) established by geodetic surveys and optical levelings or transits, h is the height of water above the NGVD, h_o is the hydrostatic water height, f is the Coriolis parameter, x_τ and y_τ are the components of surface stresses, and $A_r, A_i, B_r, B_i, C_r,$ and C_i are the bottom stress terms [Jelesnianski et al. 1992]

The surface stress, $\vec{\tau}$, is an important term in the equations of motion. Generally, the wind stress per unit mass on the sea surface is expressed as:

$$\vec{\tau}_{(x,y,t)} = C_D \frac{\rho_a}{\rho_w} |\vec{W}_{(x,y,t)}| \vec{W}_{(x,y,t)}$$

where C_D is the drag coefficient, ρ_w and ρ_a are densities of water and air, and W is the vector wind. The z coordinate of the stress term is $z = z_s$ where z_s is the distance above the sea surface typically 10 meters and where meteorological wind sources retained at the surface utilize a constant pressure to be converted to z_s [Jelesnianski et al. 1992]. Rather than the vector wind

Generalized Wave Continuity Equation (GWCE)

- ADCIRC uses the finite element method
- The primitive continuity equation (prognostic version) gives inaccurate solutions for 2Δx modes using FE method. Improve the numerical properties with a second derivative relationship $\frac{\partial^2 \eta}{\partial t^2}$

GWCE = Generalized Wave Continuity Equation

– Manipulation of governing Shallow Water Equations (SWE)

$$\frac{\partial(PCE)}{\partial t} + \tau_0(PCE) - \nabla \cdot M_c = 0$$

where *PCE* is the primitive continuity equation and M_c is the conservative momentum equation

A parameter controls the relative weight of the primitive continuity equation,

$\tau_0 \rightarrow 0$ Pure wave equation

$\tau_0 \rightarrow \infty$ Pure continuity equation

- This is the GCWE equation. For FE solutions, it has excellent numerical amplitude and phase propagation characteristics.
- ADCIRC performs storm surge simulations by solving the GCWE in combination with the momentum prognostic equations (instead of the standard continuity equation).

ADCIRC 2D Equations

Notation "musical chairs"

ADCIRC "ζ " same as class η
ADCIRC "H" is h+ ζ

ADCIRC "***q***" is product of water velocity times depth

ADCIRC "η" related to Newtonian equilibrium tide potential

Grid

Finite element, higher resolution in area of interest

Generalized Wave Continuity Equation

$$\frac{\partial^2\zeta}{\partial t^2}+\tau_0\frac{\partial\zeta}{\partial t}-\mathbf{q}\nabla\cdot\tau_0-\nabla\cdot\left\{\nabla\cdot(\mathbf{q}v)+\mathbf{f}\times\mathbf{q}+\tau_0\mathbf{q}+H\nabla\left[\frac{p_a}{\rho_0}+g(\zeta-\alpha\eta)\right]-\frac{\boldsymbol{\tau}_s+\boldsymbol{\tau}_b}{\rho_0}-\varepsilon\nabla^2(\mathbf{q})\right\}=0$$

for $\mathbf{q}=H\mathbf{v}$

Non-conservative Momentum Equation

$$\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\nabla\cdot(\mathbf{v})+\mathbf{f}\times\mathbf{v}+\nabla\left[\frac{p_a}{\rho_0}+g(\zeta-\alpha\eta)\right]-\frac{\boldsymbol{\tau}_s+\boldsymbol{\tau}_b}{\rho_0 H}-\frac{\varepsilon}{H}\nabla^2(H\mathbf{v})=0$$

Acceleration

Advection

Coriolis

Atmos. Pressure

Gravity

Tides

Surface stress wind/waves

Bottom Stress

Momentum dispersion

ADCIRC Momentum Equations, viewed another way

6. Contribution of Physical Components

[73] The shallow water momentum equation can be described in terms of its components (L: local acceleration, A: advection, C: Coriolis, Z: surface gradient, P: atmospheric pressure, T: tidal potential, W: wind stress, R: wave radiation stress gradient, B: bottom stress, D: diffusion):

$$\begin{aligned} 0 = &-\underbrace{\frac{\partial \mathbf{u}}{\partial t}}_{L} - \underbrace{\mathbf{u} \cdot \nabla \mathbf{u}}_{A} - \underbrace{f \times \mathbf{u}}_{C} - \underbrace{g \nabla \zeta}_{Z} - \underbrace{\frac{\nabla p_s}{\rho_0}}_{P} \\ &+ \underbrace{g \nabla \alpha \eta}_{T} + \underbrace{\frac{\tau_{\mathbf{s,winds}}}{\rho_0 H}}_{W} + \underbrace{\frac{\tau_{\mathbf{s,waves}}}{\rho_0 H}}_{R} - \underbrace{\frac{\tau_{\mathbf{b}}}{\rho_0 H}}_{B} + \underbrace{\frac{\mathbf{M}}{H}}_{D} \end{aligned} \tag{13}$$

where $\mathbf{u}$ represents the depth average velocity, f is the coriolis term, ζ represents the free surface departure from the geoid, p_s represent the atmospheric pressure at the sea surface, α is the earth elasticity reduction factor, η is the Newtonian equilibrium tide potential, $\tau_{s,\text{winds}}$ and $\tau_{s,\text{waves}}$ represent the imposed surface stresses for winds and waves respectively, τ_b represents the bottom stress, and M represents lateral stress gradients.

Wave setup references

Bowen, A. J., Inman, D. L., and Simmons, V. P.: Wave “set-down” and wave setup, J. Geophys. Res., 73, 2569-2577, 1968.

Dean, R. G., and Walton, T. L.: Wave Setup, Chapter 1 in Handbook of Coastal and Ocean Engineering, ed. Kim, Y. C., World Scientific Publishing Co., Hackensack, NJ, USA, 1192 pp., 2009.

Longuet-Higgins, M. S., and Stewart, R. W.: Radiation stress and mass transport in gravity waves with application of ‘surf beats’, J. Fluid Mechanics, 13, 481-504, 1962.

Longuet-Higgins, M. S., and Stewart, R. W.: Radiation stresses in water waves; a physical discussion, with applications, Deep-Sea Res., 11, 529-562, 1964.