

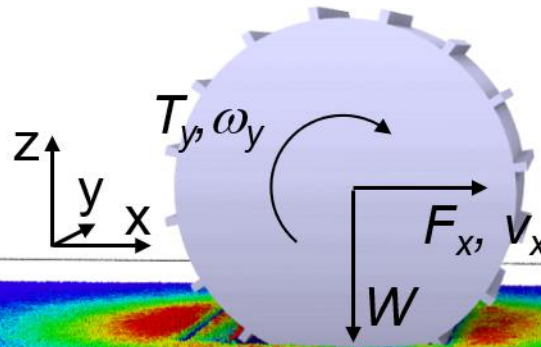
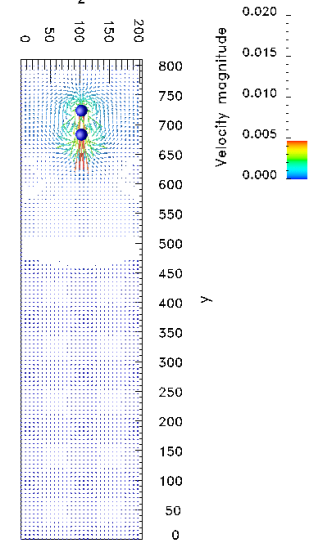
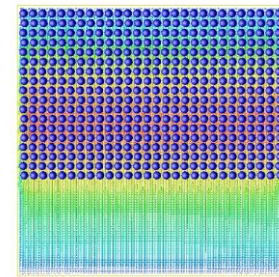
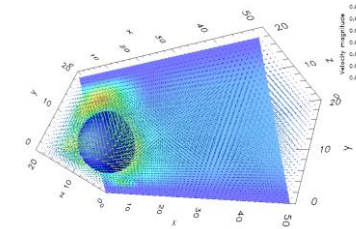
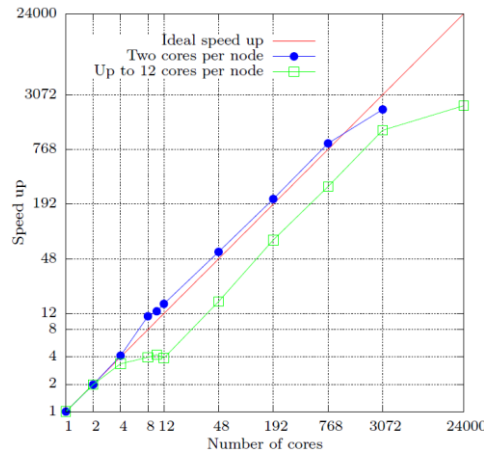
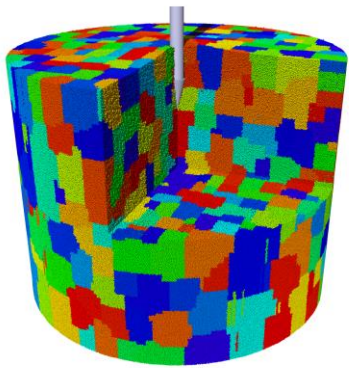
# Large-scale DEM-LBM Modeling Towards Off-road Mobility

2019 DoD HCPMP User Group Meeting

May 7-8, 2019

Vicksburg, MS

Bohumir Jelinek



MISSISSIPPI STATE  
UNIVERSITY

# Summary

Lattice-Boltzmann method (LBM) for fluid flow

Discrete Element Method (DEM) for granular media

Coupled DEM-LBM system for shear thickening

DEM simulations of Cone Penetrometer Test,  
particle size convergence study

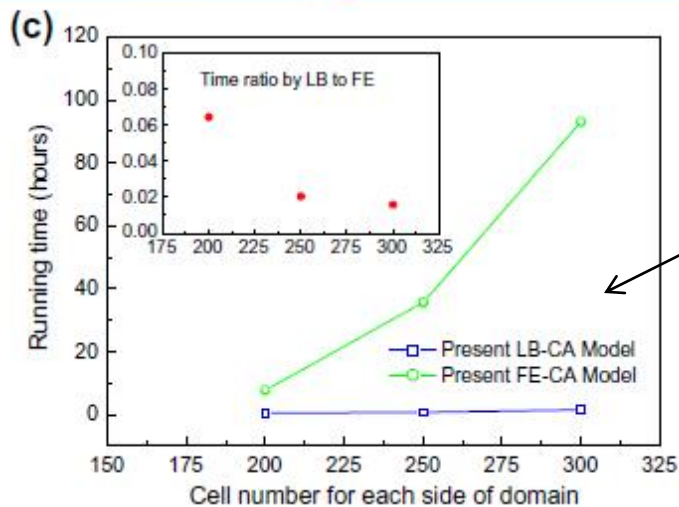
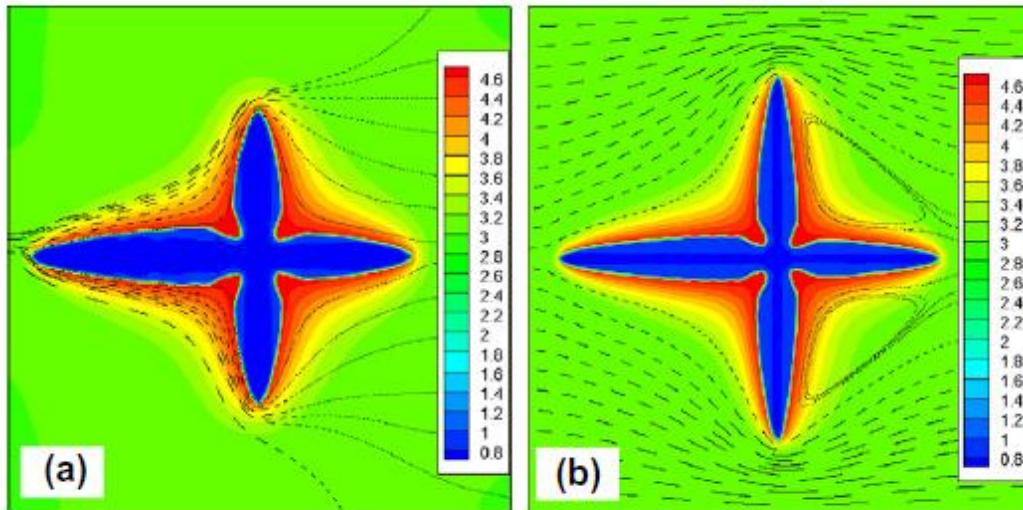
DEM simulations of small driven wheel

DEM simulations to obtain pull/traction-slip curve  
for smallest wheel in the DROVE database

DEM simulations of low-deflection wheel

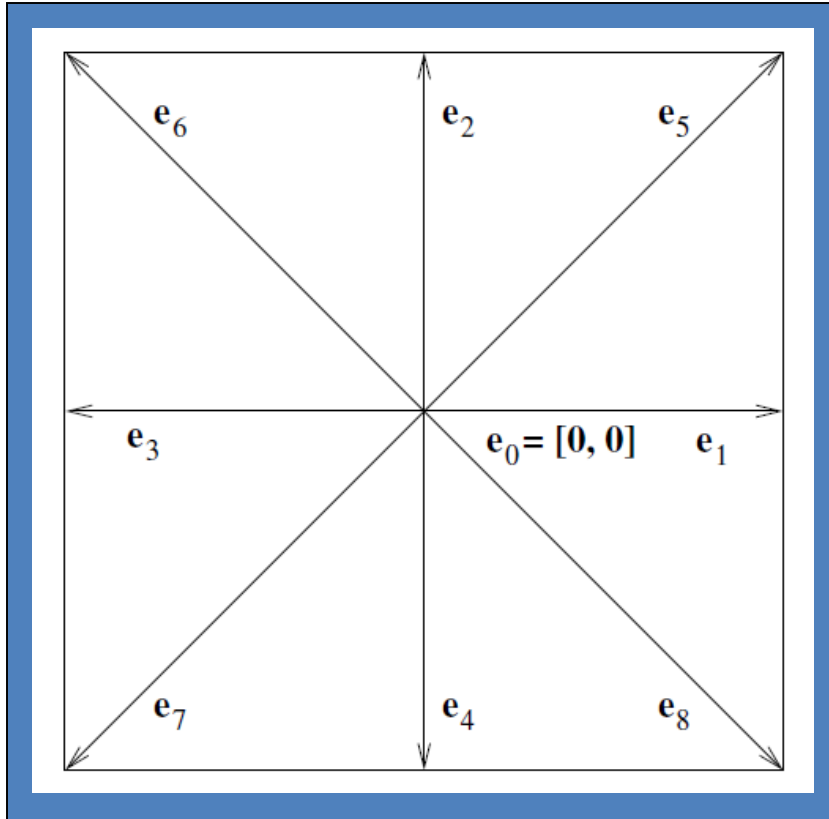
# Why LBM/CA?

*H. Yin et al. / Acta Materialia 59 (2011) 3124–3136*



For the full resolution calculation of the fluid flow around solidifying dendrites, Lattice-Boltzmann / Cellular Automata method is faster than alternatives.

# Lattice-Boltzmann method



## D2Q9 lattice

Each node has 9 distribution functions  $f_i$  representing portion

of the mass density moving in the lattice direction  $\mathbf{e}_i$

$$\rho = \sum_{i=0}^8 f_i, \quad \rho \mathbf{u} = \sum_{i=0}^8 f_i \mathbf{e}_i$$

$$f_i(\mathbf{r} + \mathbf{e}_i \Delta t, t + \Delta t) = f_i(\mathbf{r}, t) + \frac{1}{\tau_u} \left( f_i^{\text{eq}}(\mathbf{r}, t) - f_i(\mathbf{r}, t) \right)$$

# Time evolution of the distribution functions

For each lattice direction  $\mathbf{e}_i$ ,  $i=0..8$

$$\nu = \frac{\tau_u - 0.5 \frac{\Delta x^2}{\Delta t}}{3}$$

$\tau_u$  ~kin. viscosity

$$f_i(\mathbf{r} + \mathbf{e}_i \Delta t, t + \Delta t) = f_i(\mathbf{r}, t) + \frac{1}{\tau_u} \left( f_i^{\text{eq}}(\mathbf{r}, t) - f_i(\mathbf{r}, t) \right)$$

Streaming:

Shifts each distribution function to the neighboring node

Collision:

Adjusts the distribution function to approach equilibrium distribution

# Equilibrium distribution function

$$f_i^{\text{eq}}(\mathbf{r}) = w_i \rho(\mathbf{r}) \left( 1 + 3 \frac{\mathbf{e}_i \cdot \mathbf{u}(\mathbf{r})}{c^2} + \frac{9}{2} \frac{(\mathbf{e}_i \cdot \mathbf{u}(\mathbf{r}))^2}{c^4} - \frac{3}{2} \frac{\mathbf{u}(\mathbf{r}) \cdot \mathbf{u}(\mathbf{r})}{c^2} \right)$$

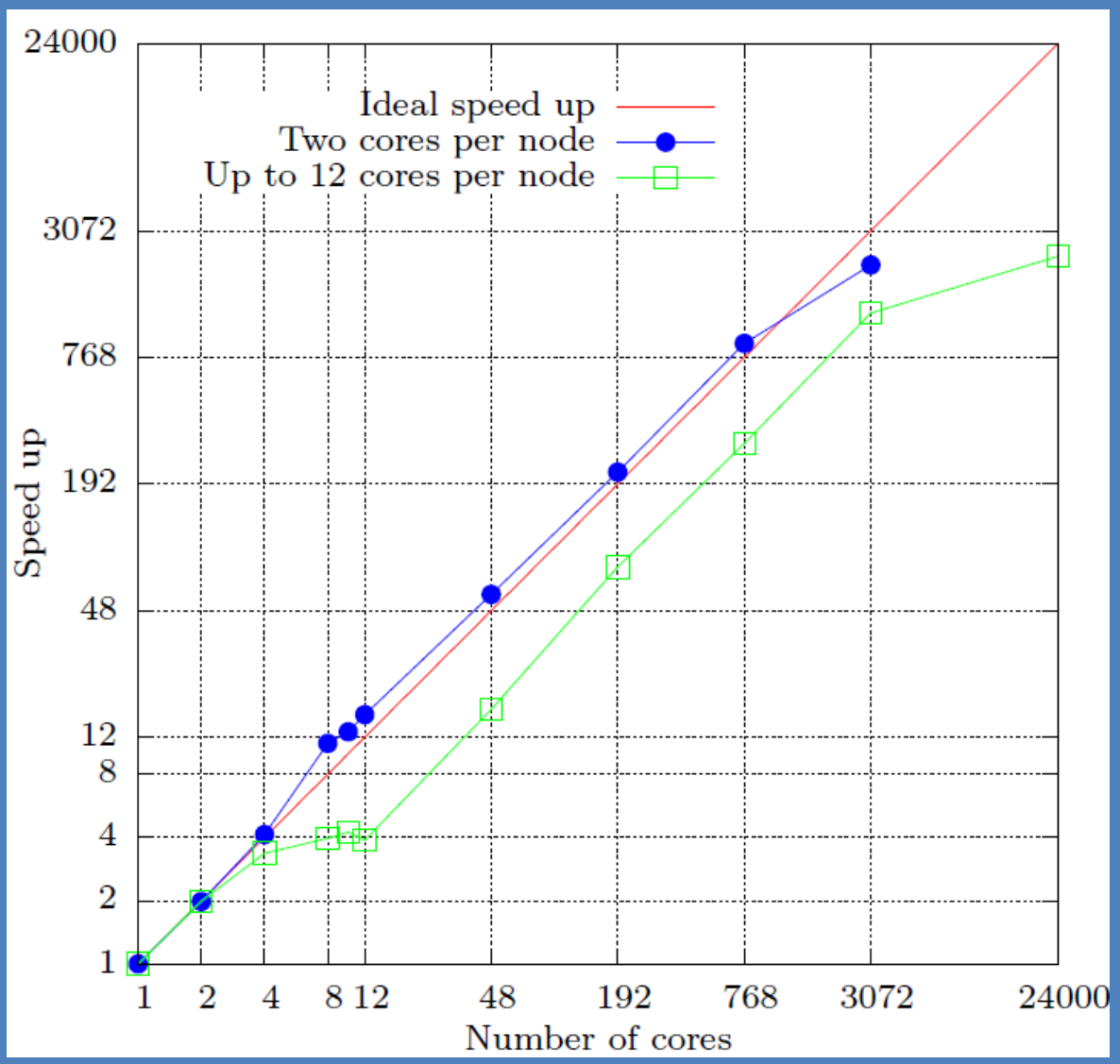


$$w_i = \begin{cases} 4/9 & i = 0 \\ 1/9 & i = 1, 2, 3, 4 \\ 1/36 & i = 5, 6, 7, 8 \end{cases}$$



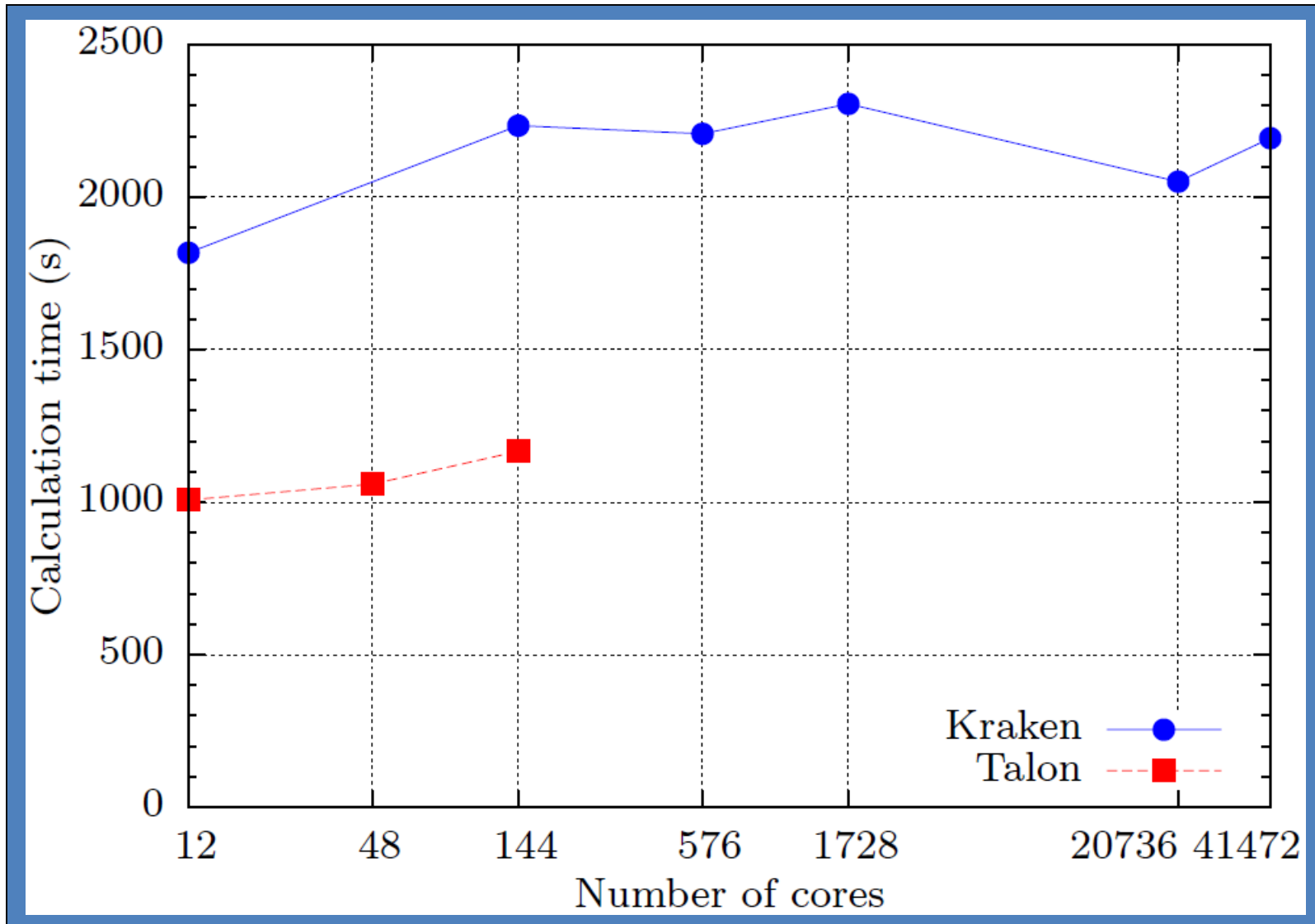
lattice velocity  $c = \Delta x / \Delta t$

# Speed up



- strong scaling (speed up) near perfect up to 3072 cores
- Algorithm is memory bandwidth limited on multi-core architecture (low FLOP/byte ratio)

# Scale up



Goal:  
constant  
calculation  
time



# Discrete Element Method

- Tracks individual particles in granular media
- Interactions between particles given by contact laws, an example is linear spring contact:

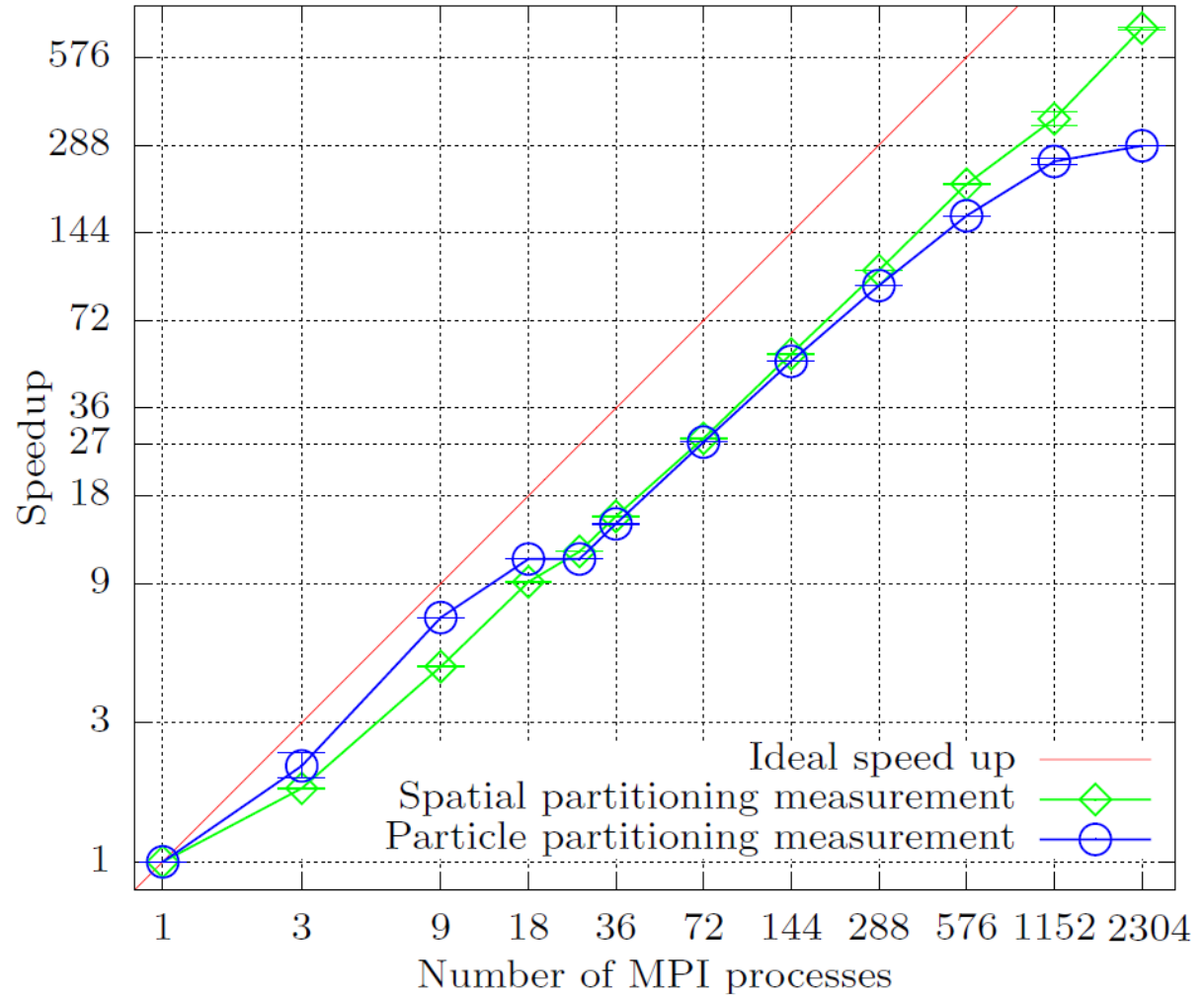
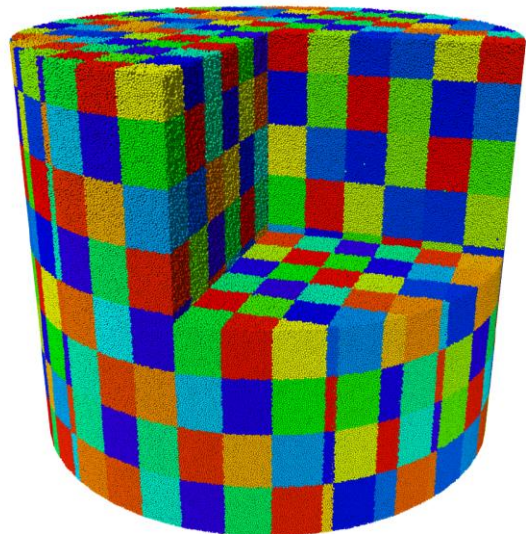
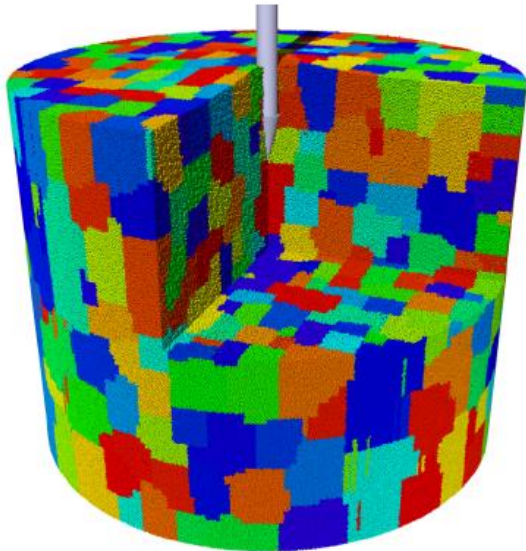
$$F_N = \alpha K_N \delta_n^m$$

- Particle linear and angular velocity are determined by integrating Newton's equations of motion:

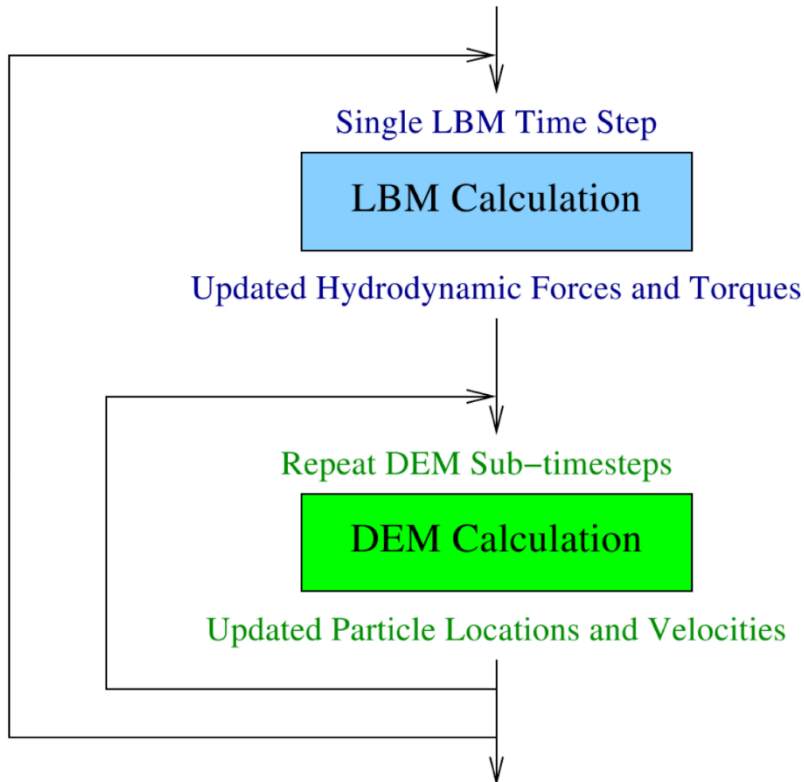
$$m \frac{\partial v_i}{\partial t} = m g_i + \sum_{c=1}^{N_c} f_i^c \quad I_m \frac{\partial \omega_i}{\partial t} = \sum_{c=1}^{N_c} M_i^c$$

# DEM parallelization

Particle (top) and spatial  
(bottom) domain decomposition



# DEM-LBM coupling algorithm

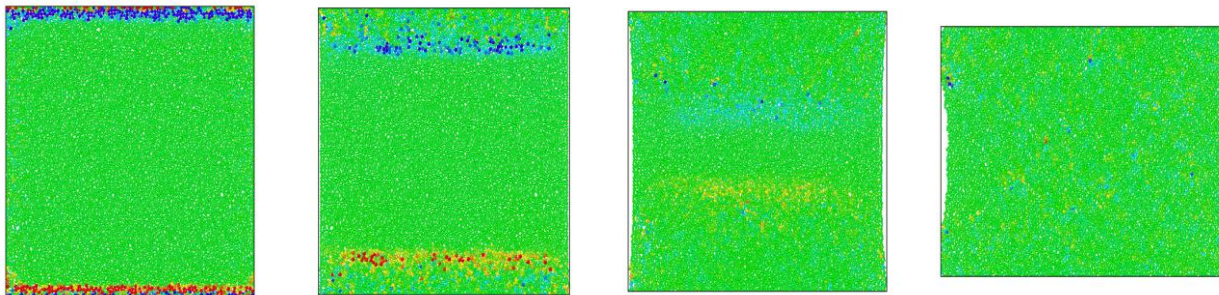
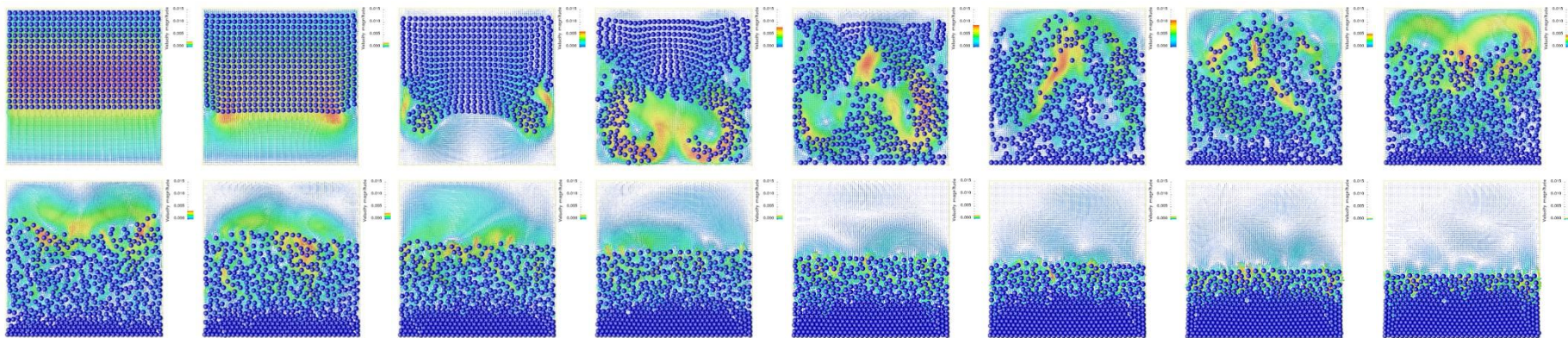
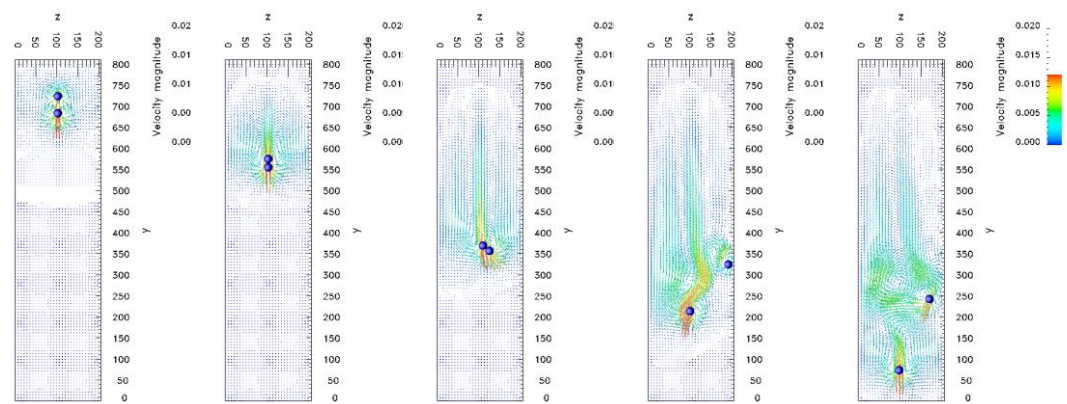
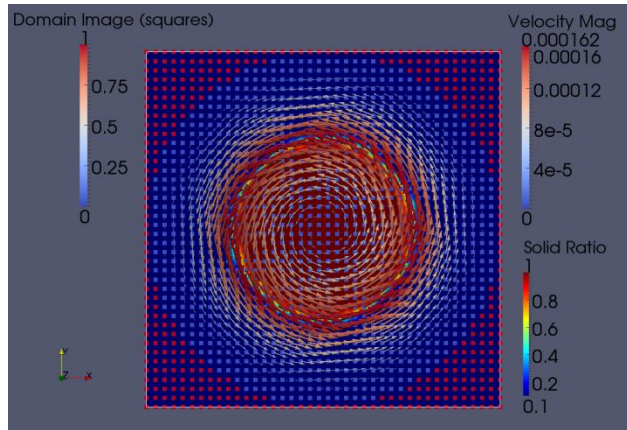


Force & torque by fluid on particles:

$$\mathbf{F}_F = \frac{\Delta x^3}{\Delta t} \sum_n \left( \beta_n \sum_{i=0}^{14} \boldsymbol{\Omega}_i^S \mathbf{e}_i \right)$$

$$\mathbf{T}_F = \frac{\Delta x^3}{\Delta t} \sum_n (\mathbf{r}_n - \mathbf{r}_c) \times \left( \beta_n \sum_{i=0}^{14} \boldsymbol{\Omega}_i^S \mathbf{e}_i \right)$$

# DEM+LBM Coupled Model of Densely Packed Particle assemblies in Fluid



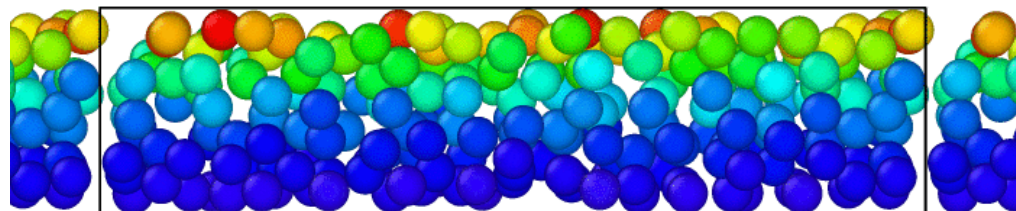
# Coupled DEM (serial) / LBM (parallel) model for saturated granular media

Starting point:

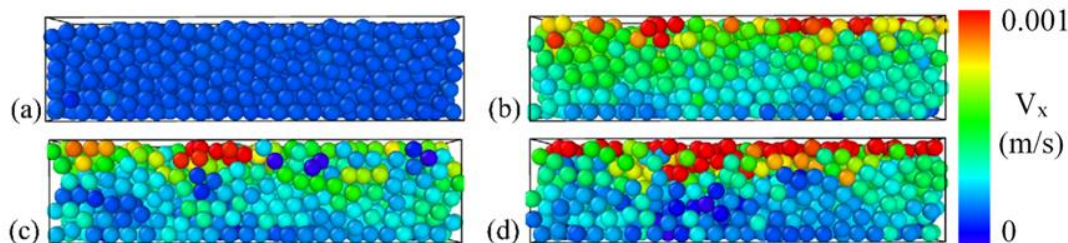
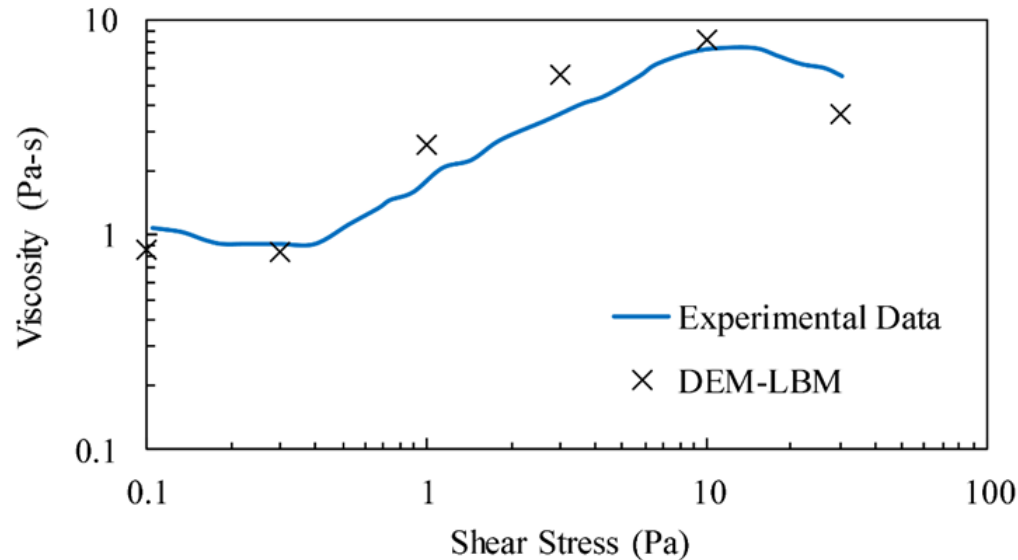
- Single-component single-phase lattice-Boltzmann model coupled with DEM for spherical particles submerged in fluid
- Coupling utilized immersed moving boundary with subgrid resolution (Owen et al., Int. J. Numer. Meth. Fluids 2007, 55)

Present application:

- Coupled DEM+LBM system for numerical modeling of shear-thickening of granular suspensions

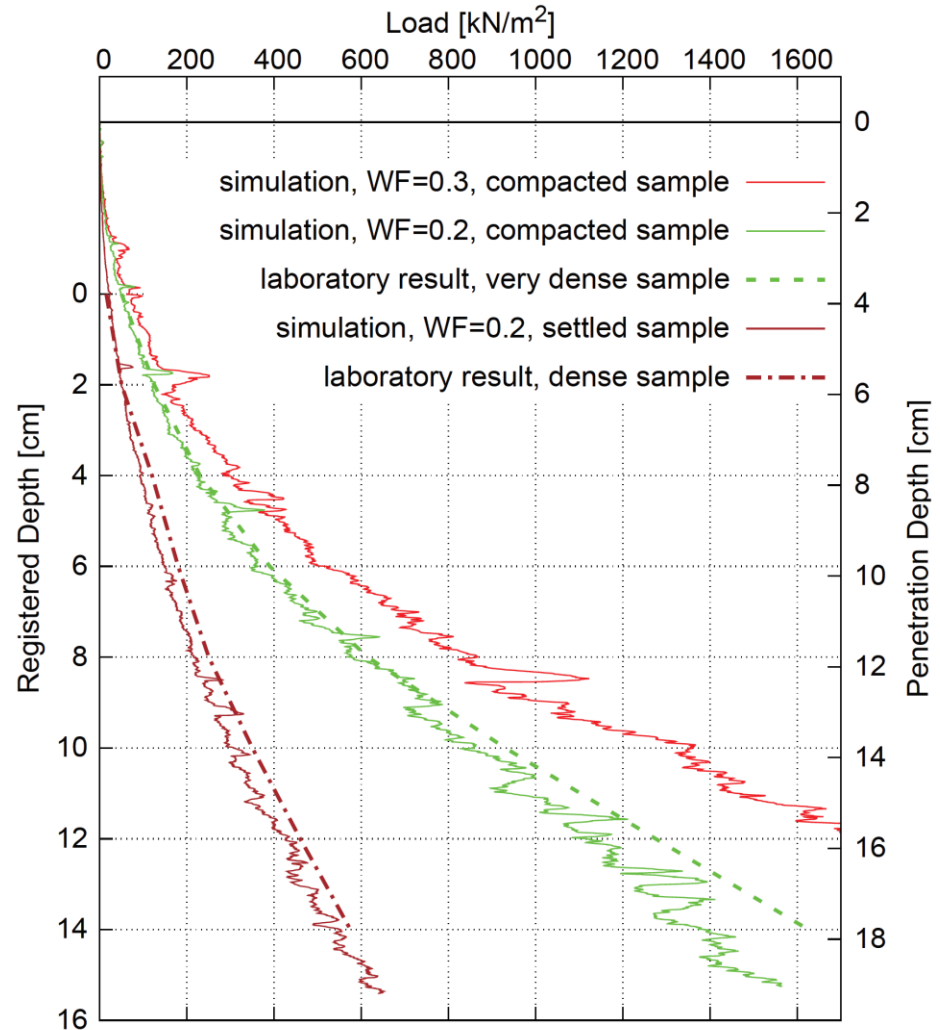
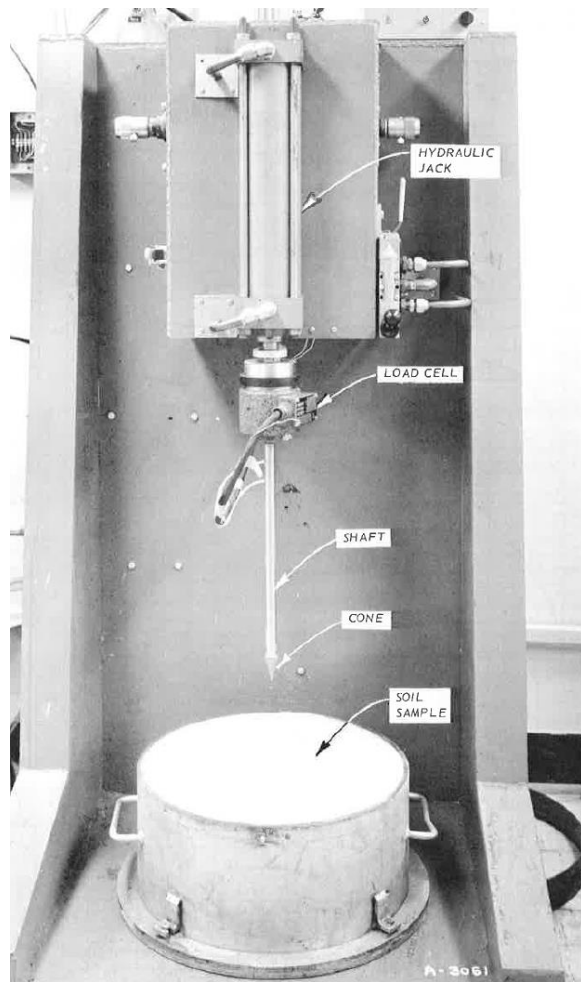


# Deformation-Fluid Coupling



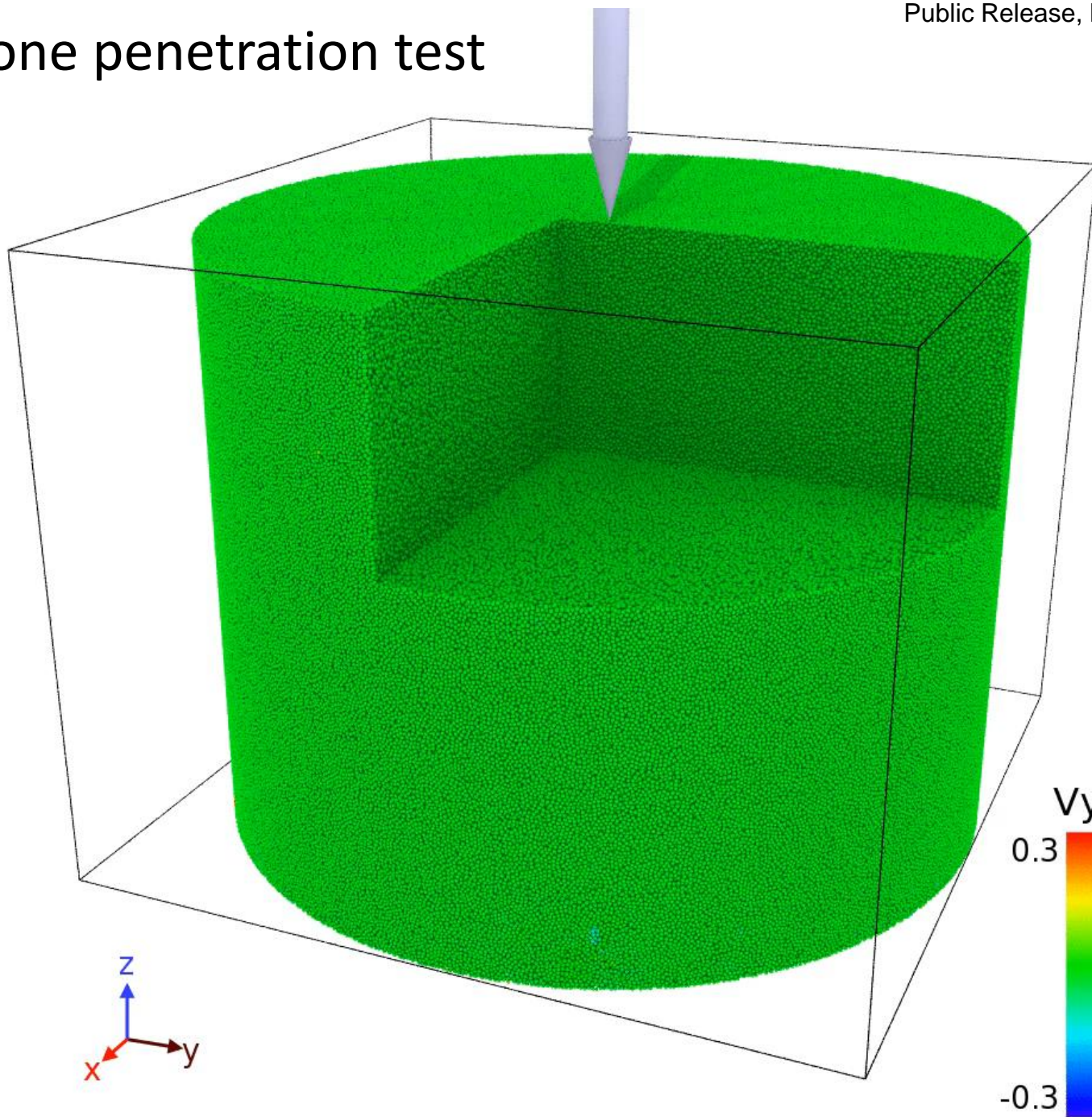
- Moisture state is an important component of subgrade reaction
- Most subgrades are partially saturated
- Successful simulation of undrained dilation is important for sand and silt subgrades
- Successful simulation of shear thickening fluids demonstrates capability to model fluid-soil interaction

# Simulation of cone penetrometer test (CPT) for calibration of DEM parameters



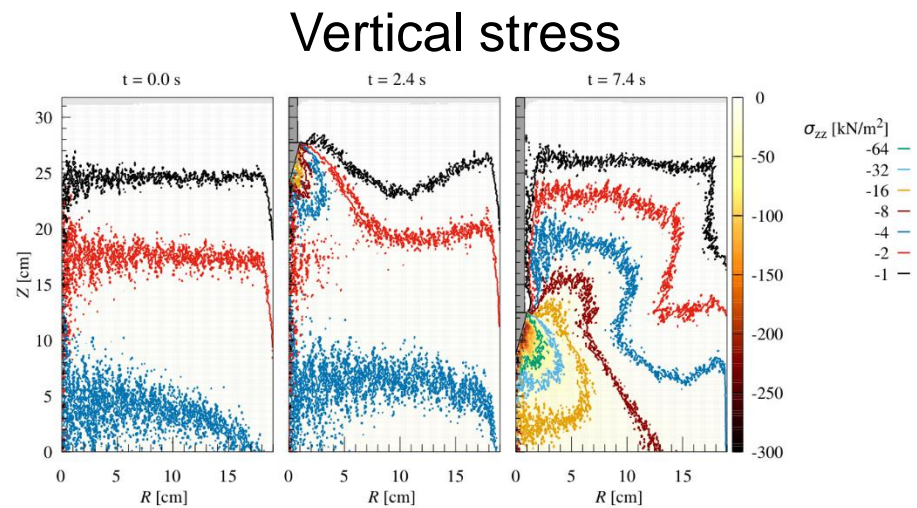
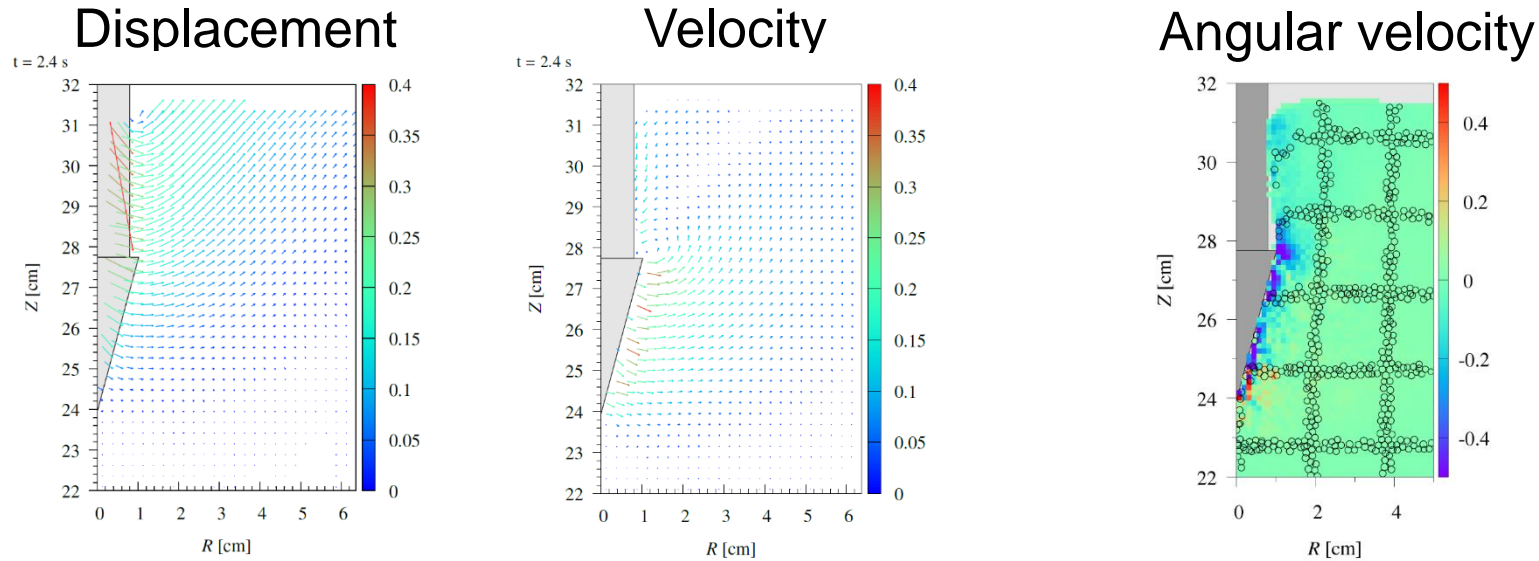
DEM: 9.5 mil. spherical particles in cylindrical mold, rigid cone-tip probe

# Cone penetration test

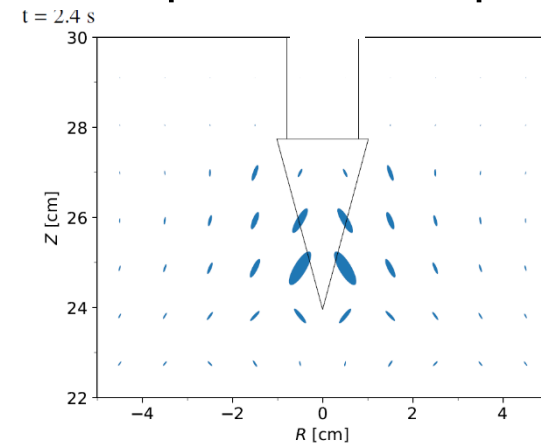




# Macroscopic averages from DEM CPT simulations



### Principal stress ellipse



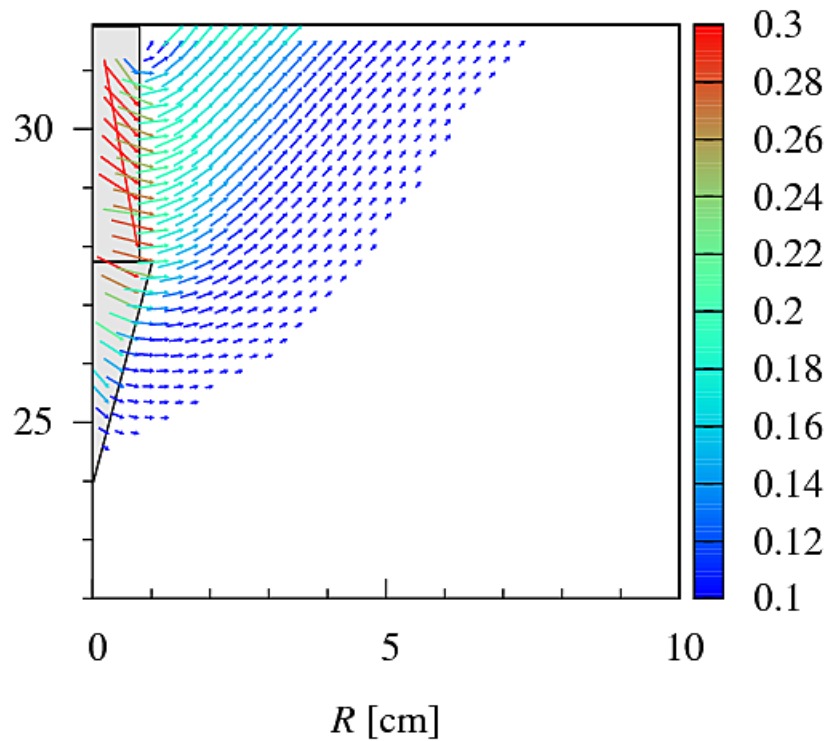
# Average particle displacement

Dense sample

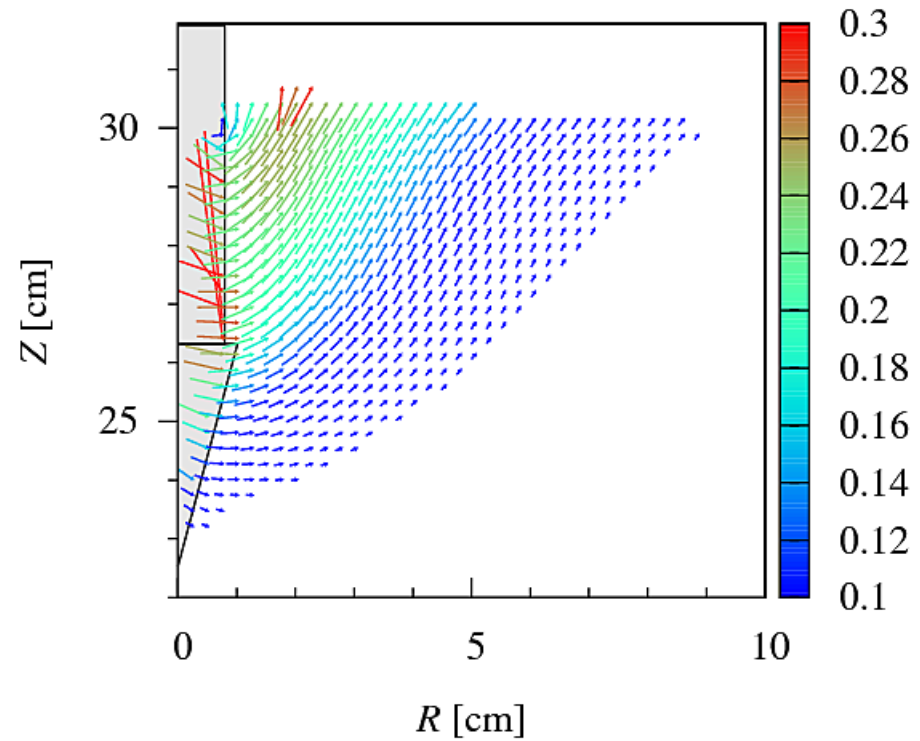
$t=2.4\text{s}$

Very dense sample

Local displacement vectors, magnitude [cm]



Local displacement vectors, magnitude [cm]



# Average particle displacement

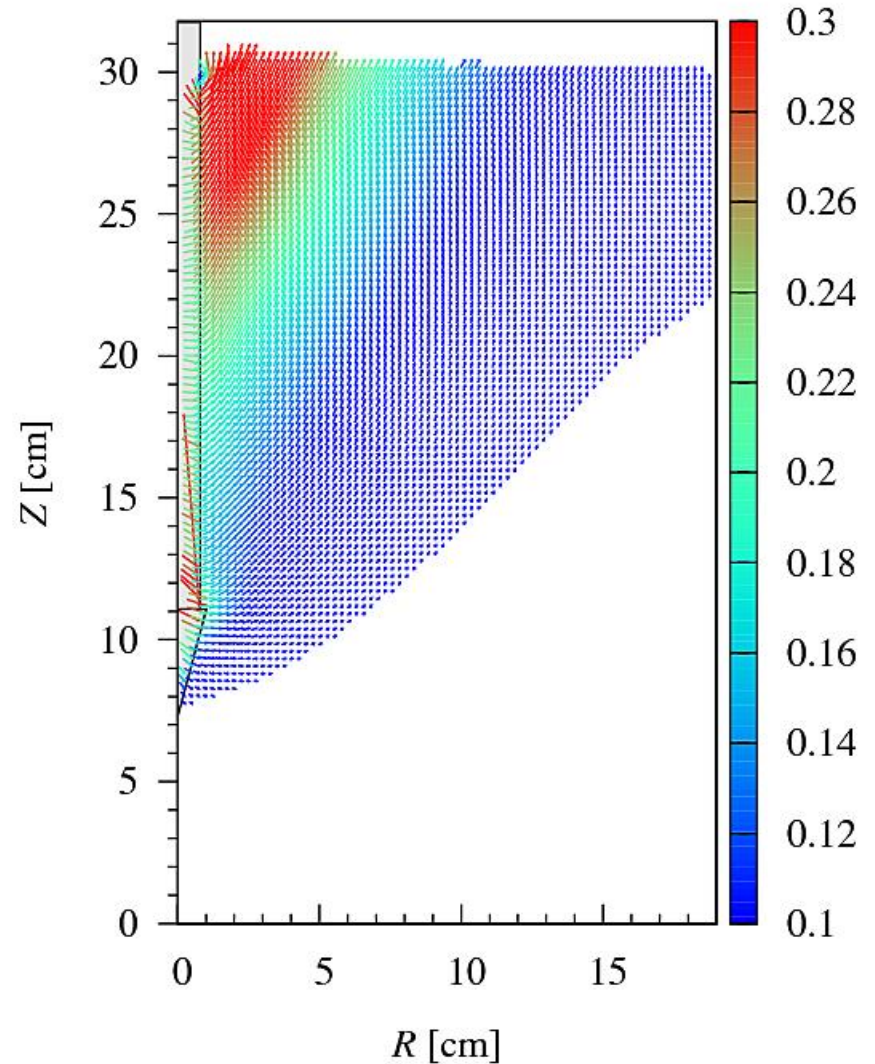
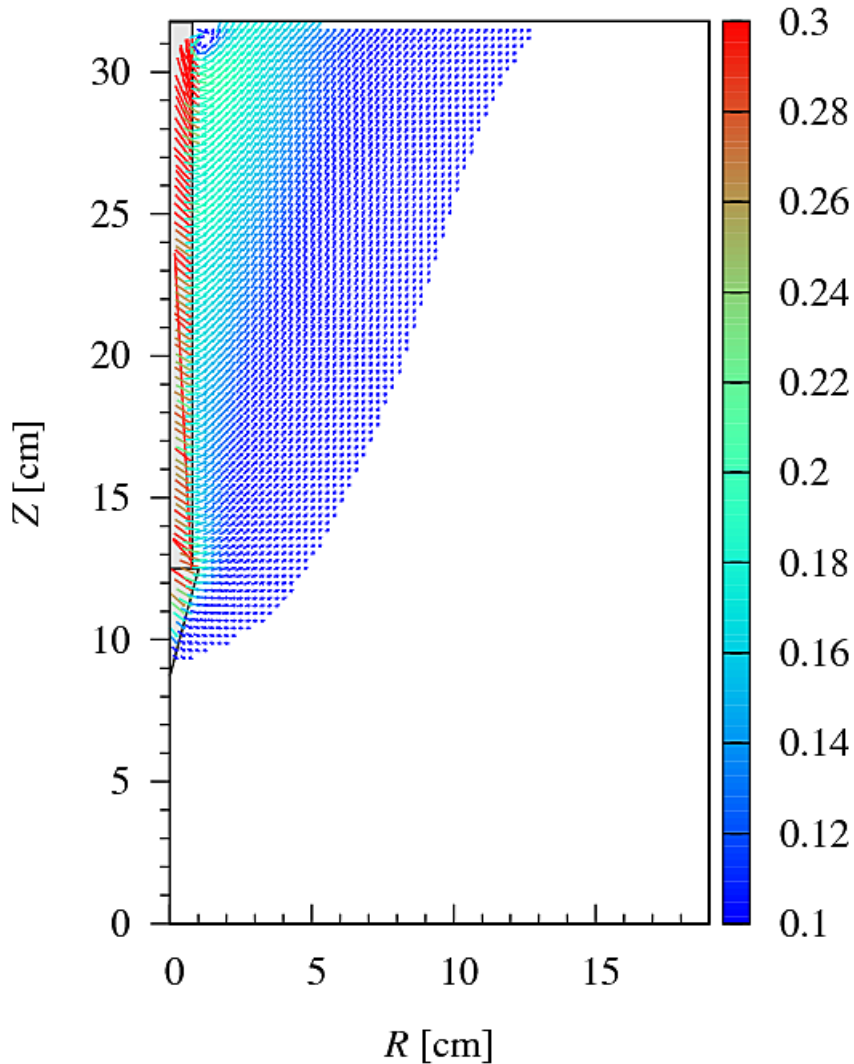
Dense sample

t=7.4s

Very dense sample

Local displacement vectors, magnitude [cm]

Local displacement vectors, magnitude [cm]



# Average particle velocities

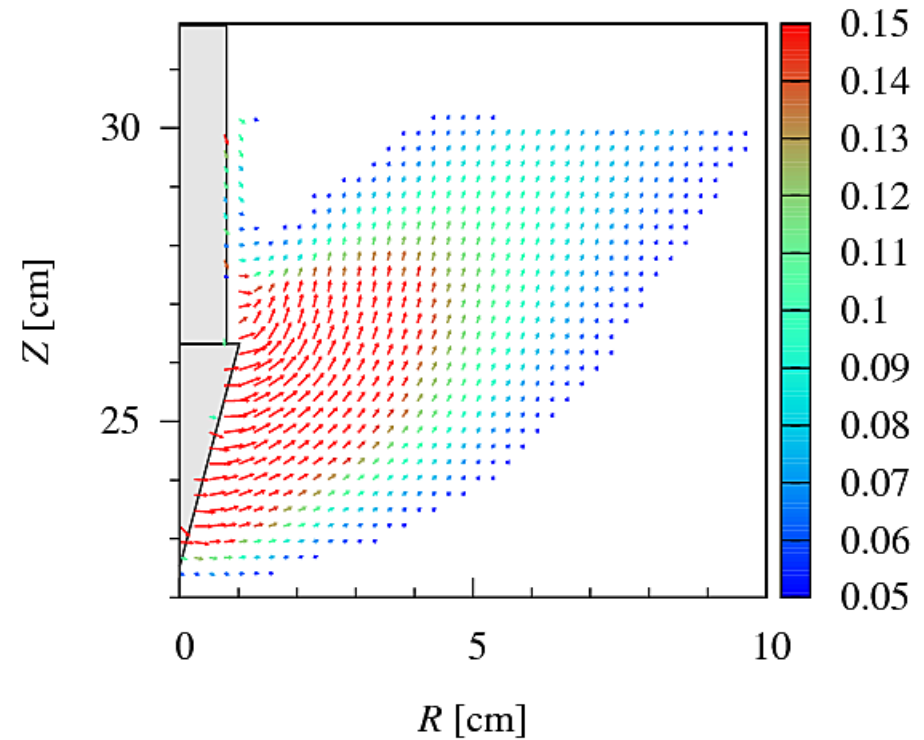
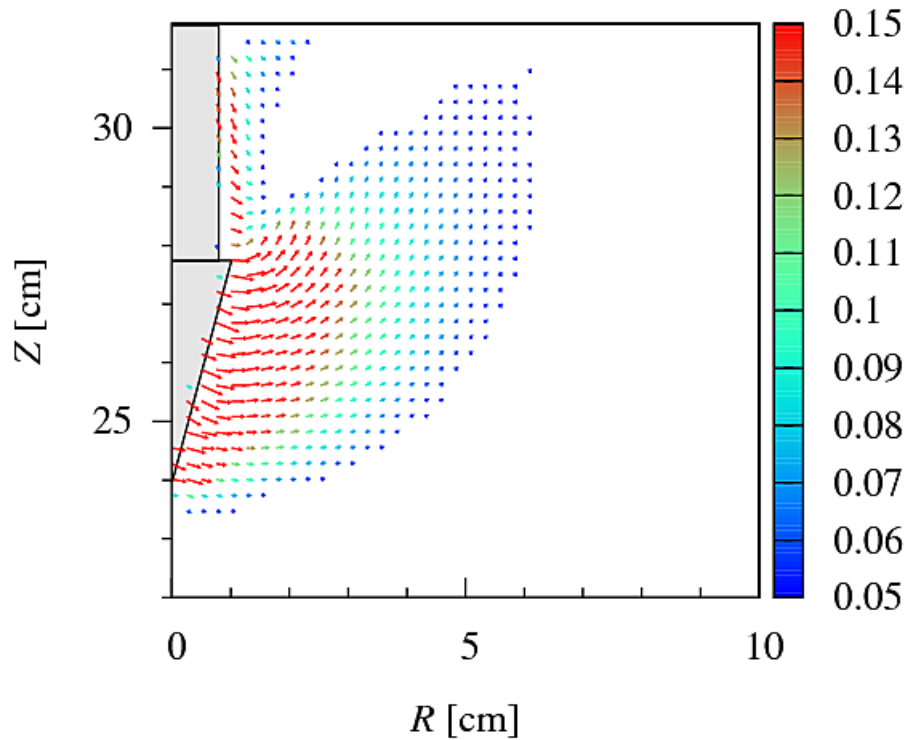
Dense sample

t=2.4s

Very dense sample

Local velocity vectors, magnitude [cm/s]

Local velocity vectors, magnitude [cm/s]



# Average particle velocities

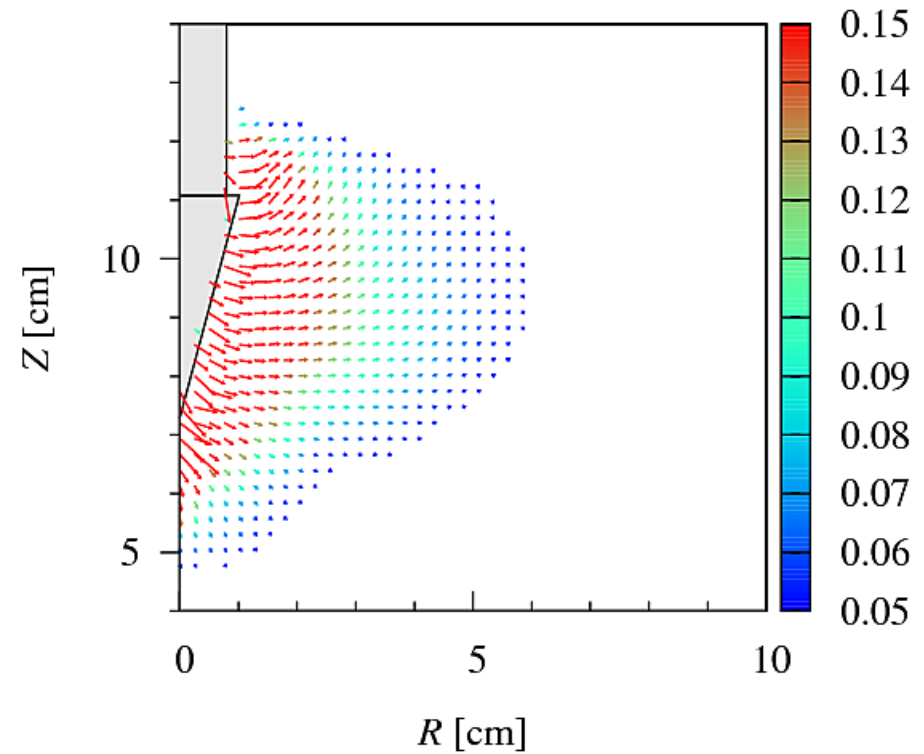
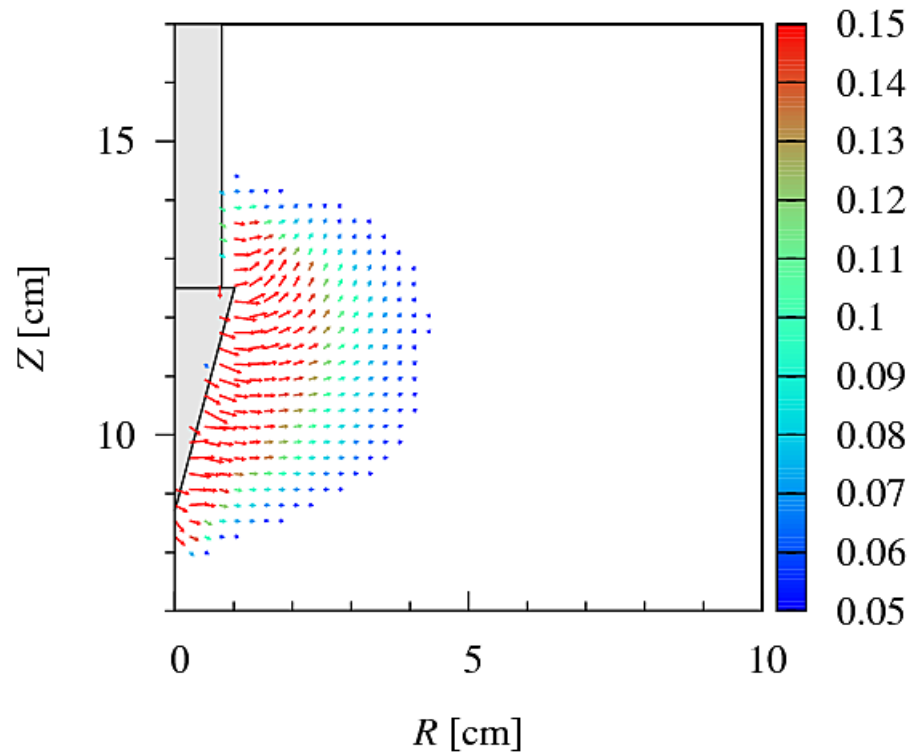
Dense sample

t=7.4s

Very dense sample

Local velocity vectors, magnitude [cm/s]

Local velocity vectors, magnitude [cm/s]

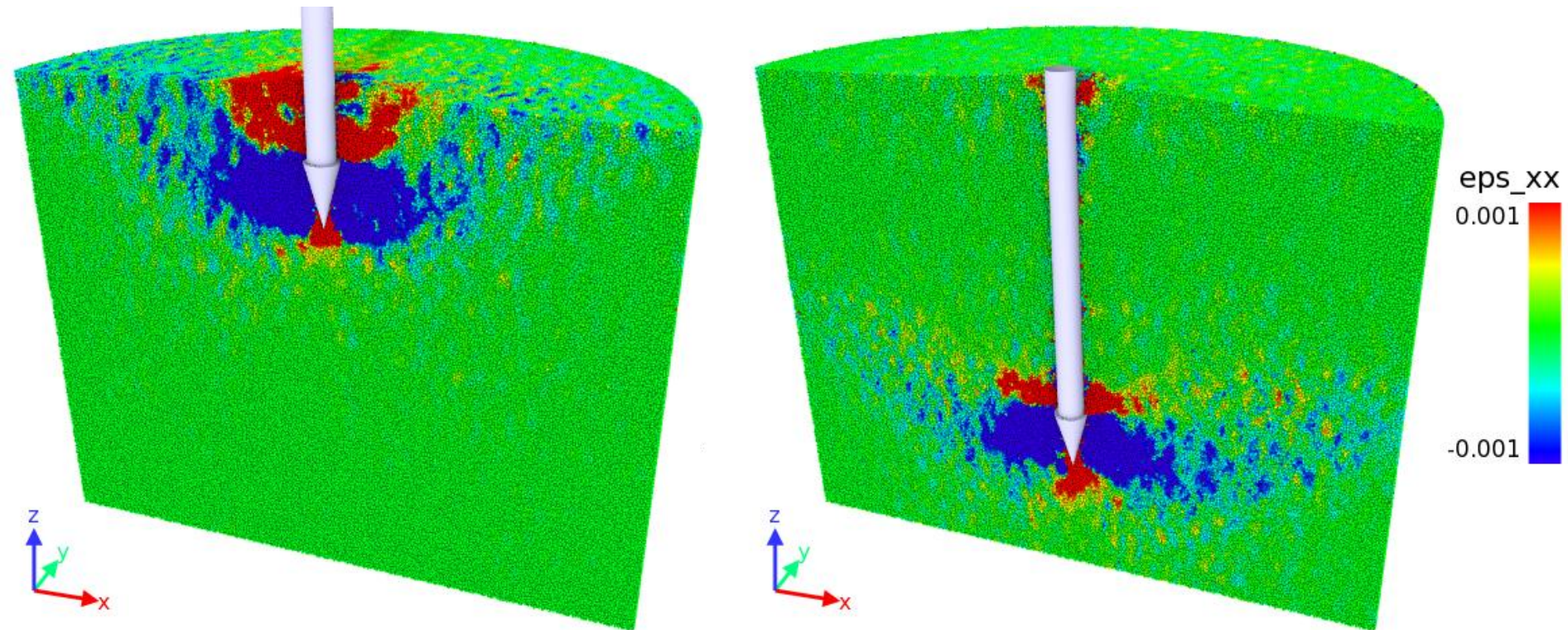


# Local strain calculated from particle displacement

t=2.4s

Very dense sample

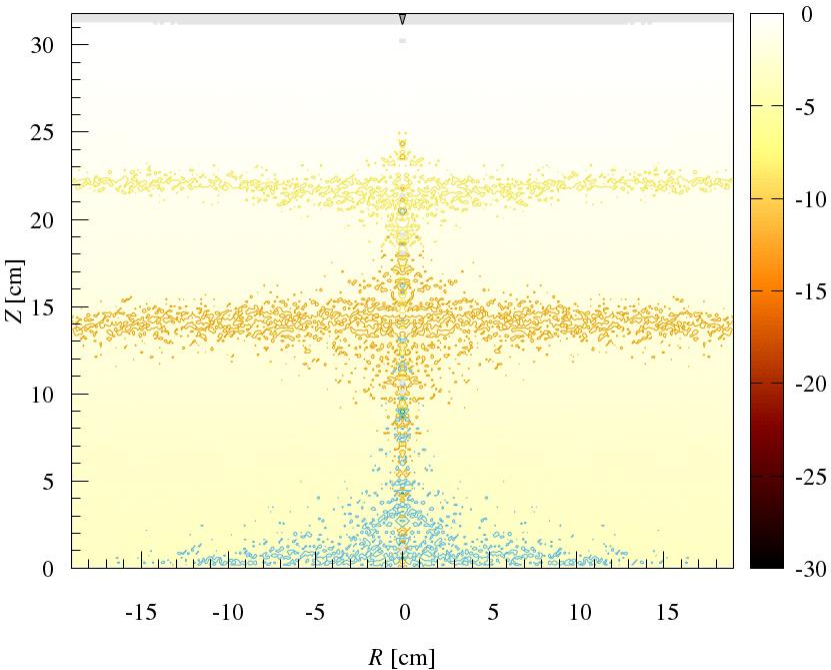
t=7.4s



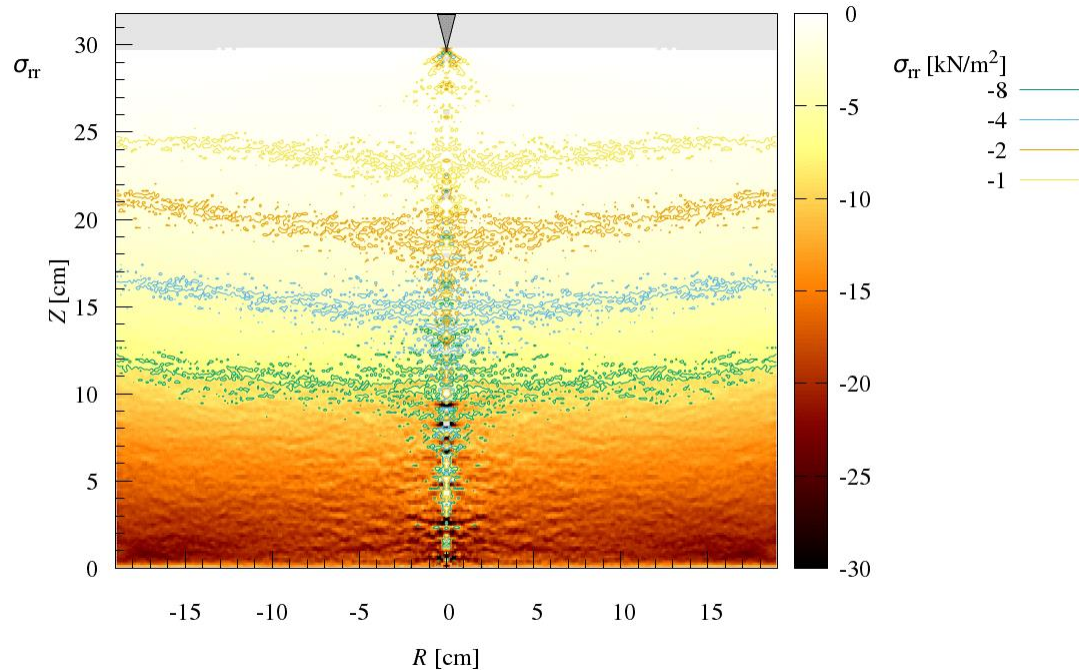
# Evaluating stress from CPT simulations

## Radial stress component $\sigma_{rr}$

Dense sample



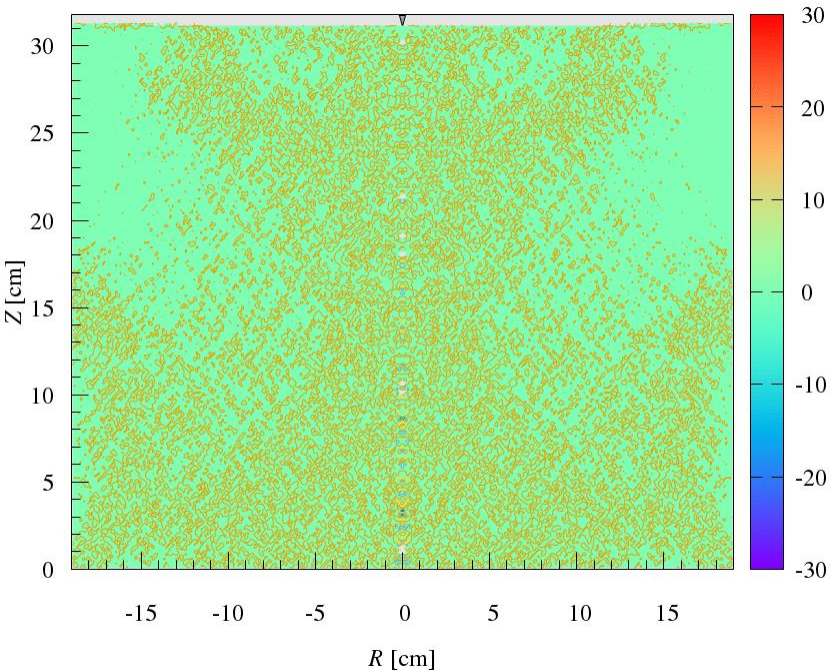
Very dense sample



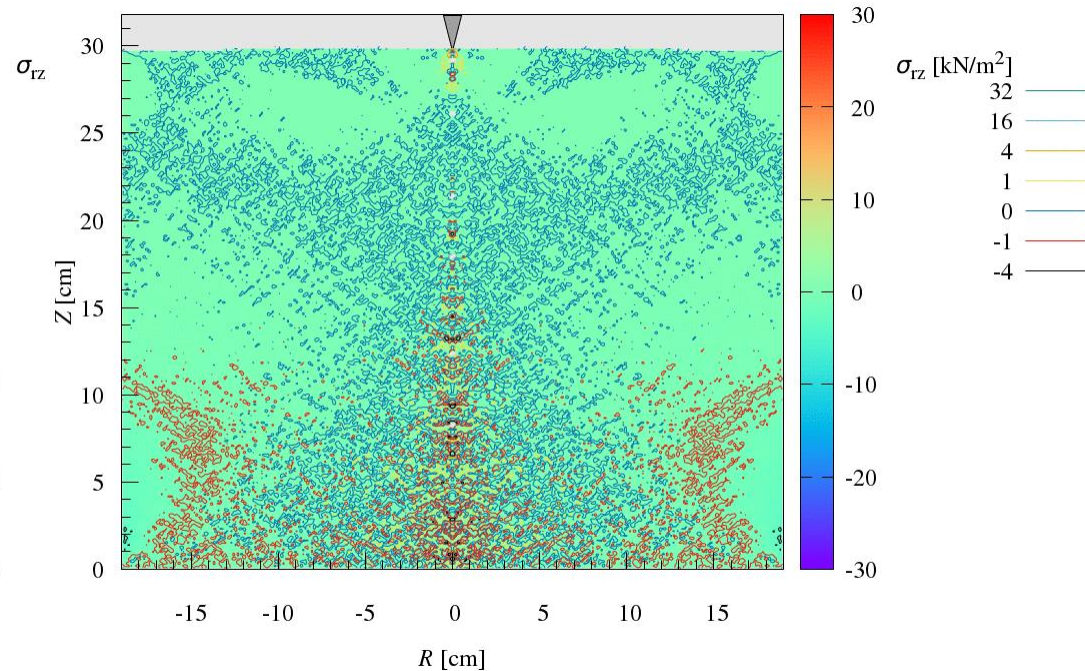
# Evaluating stress from CPT simulations

## Shear stress component $\sigma_{rz}$

Dense sample



Very dense sample



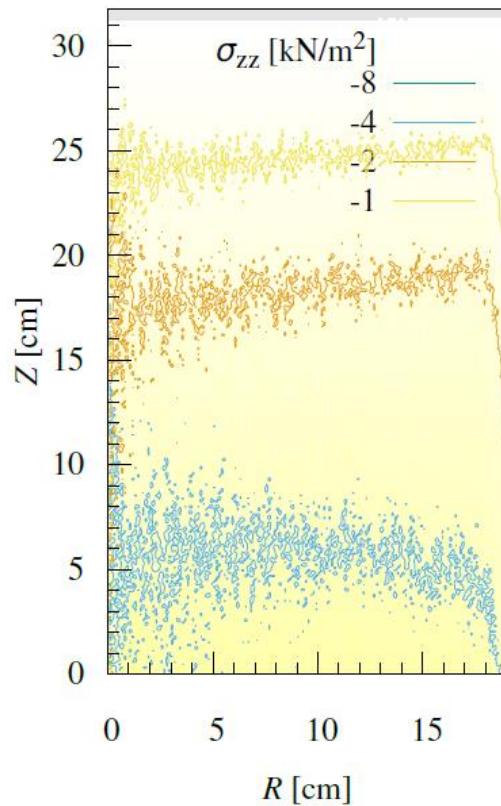


# Evaluating vertical stress during CPT

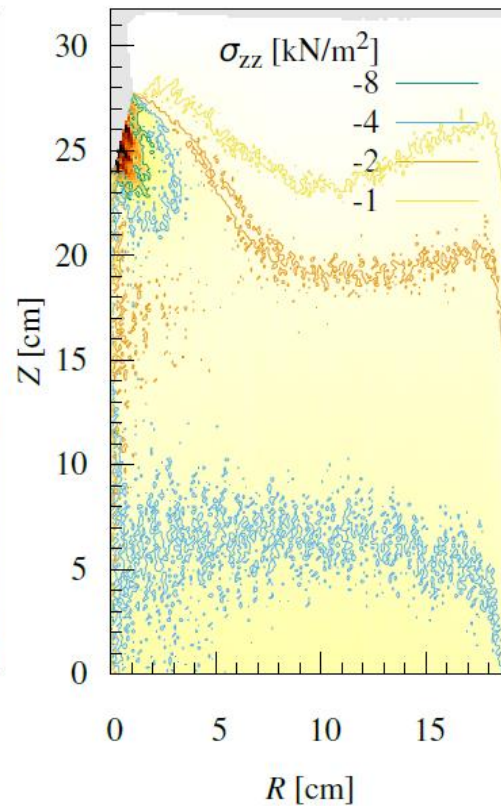
## Dense sample

### $\sigma_{zz}$ component

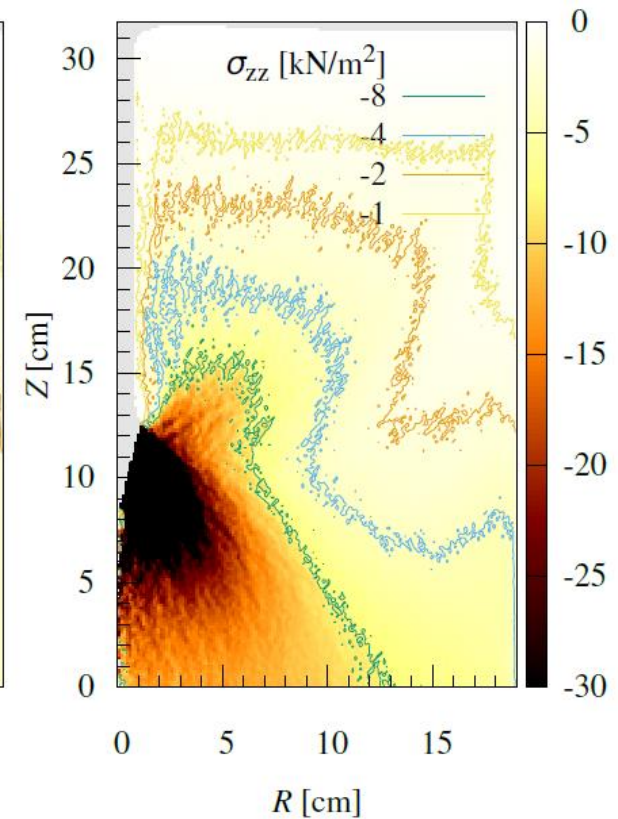
0.05 s



2.4 s



7.4 s



# Evaluating vertical stress during CPT

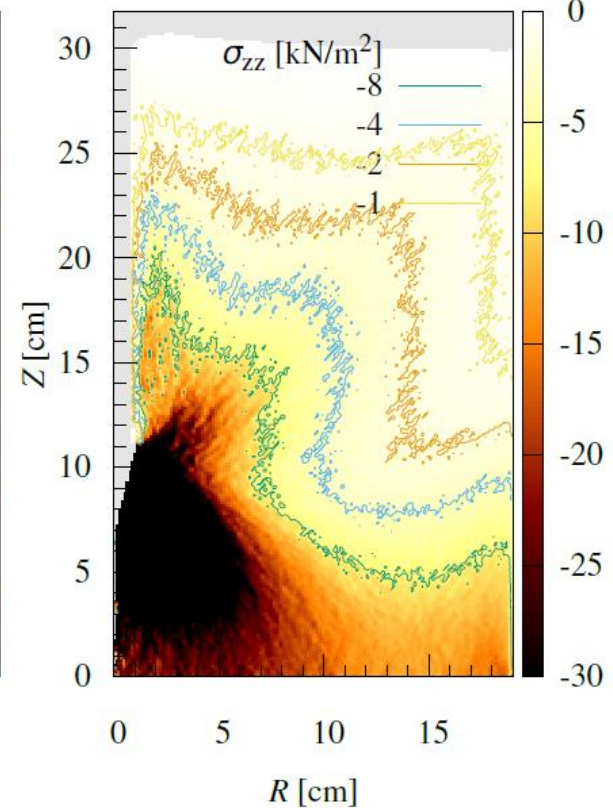
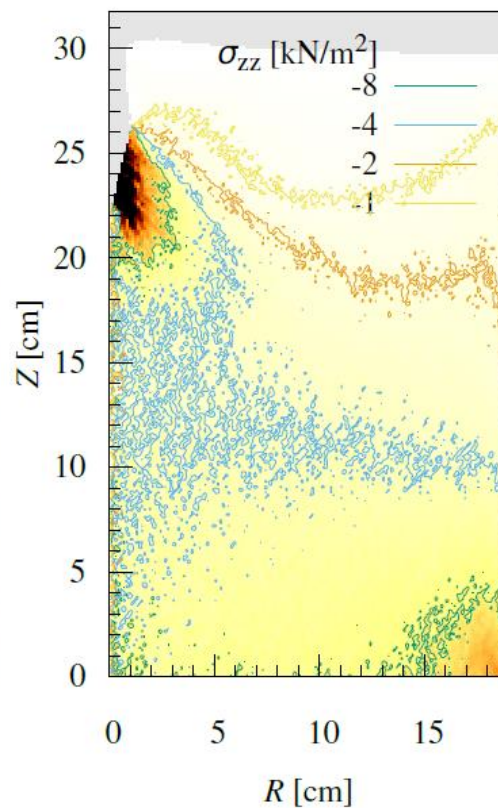
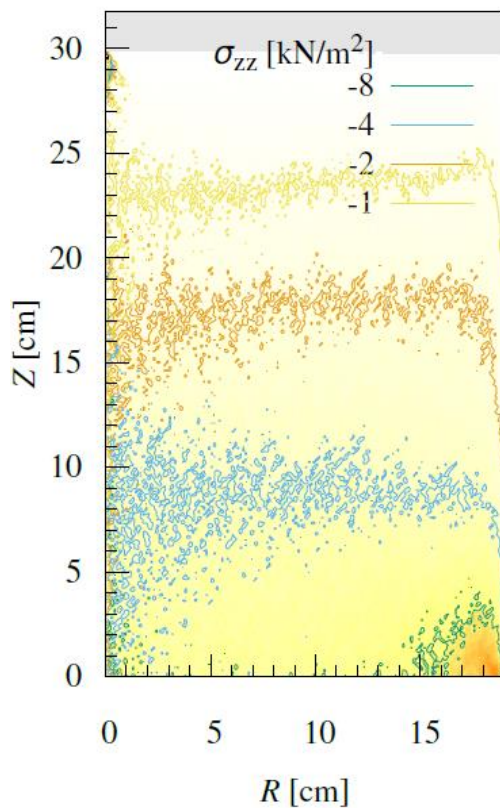
## Very dense sample

### $\sigma_{zz}$ component

0.05 s

2.4 s

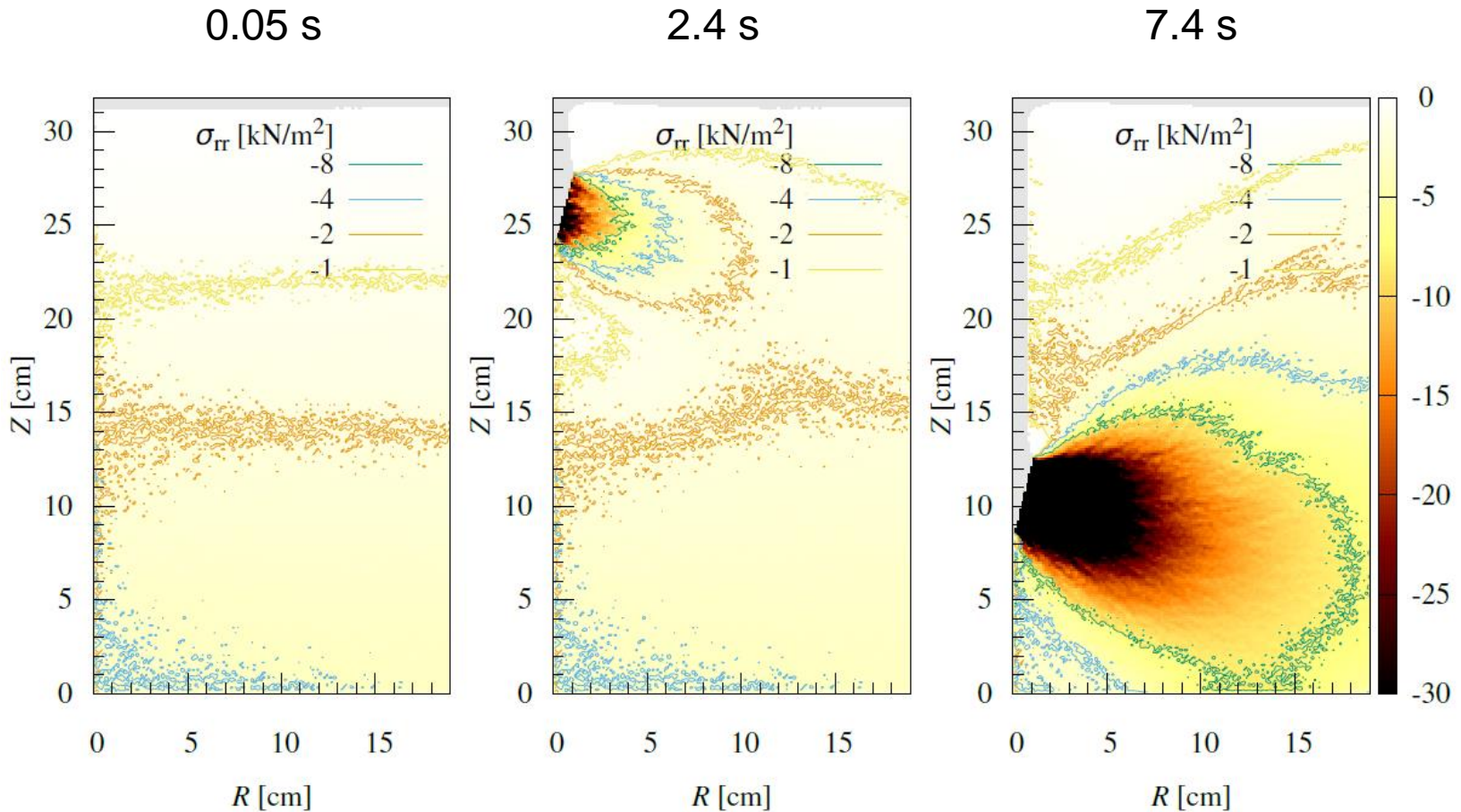
7.4 s



# Evaluating vertical stress during CPT

## Dense sample

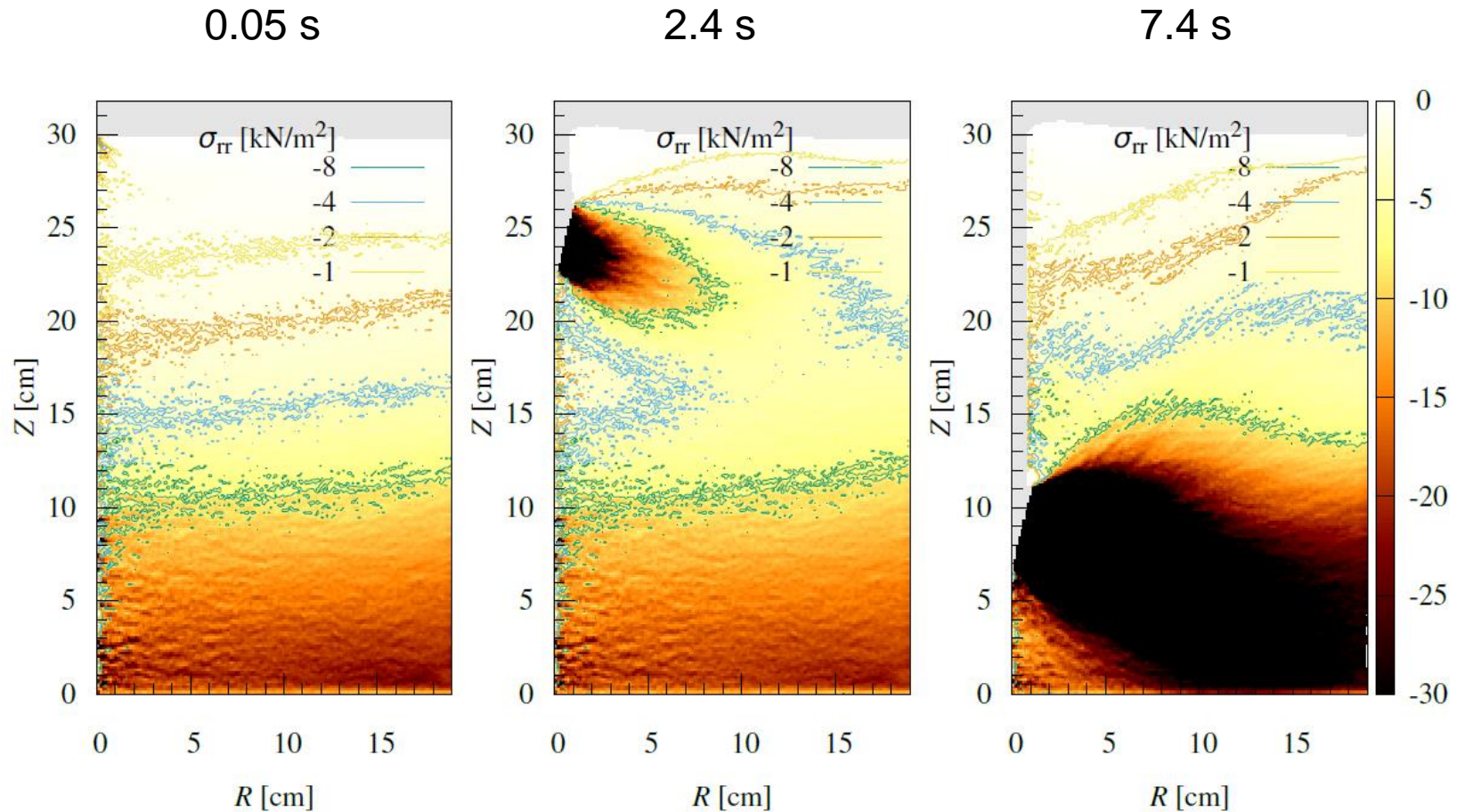
### $\sigma_{rr}$ component



# Evaluating vertical stress during CPT

## Very dense sample

### $\sigma_{rr}$ component



# Evaluating vertical stress during CPT

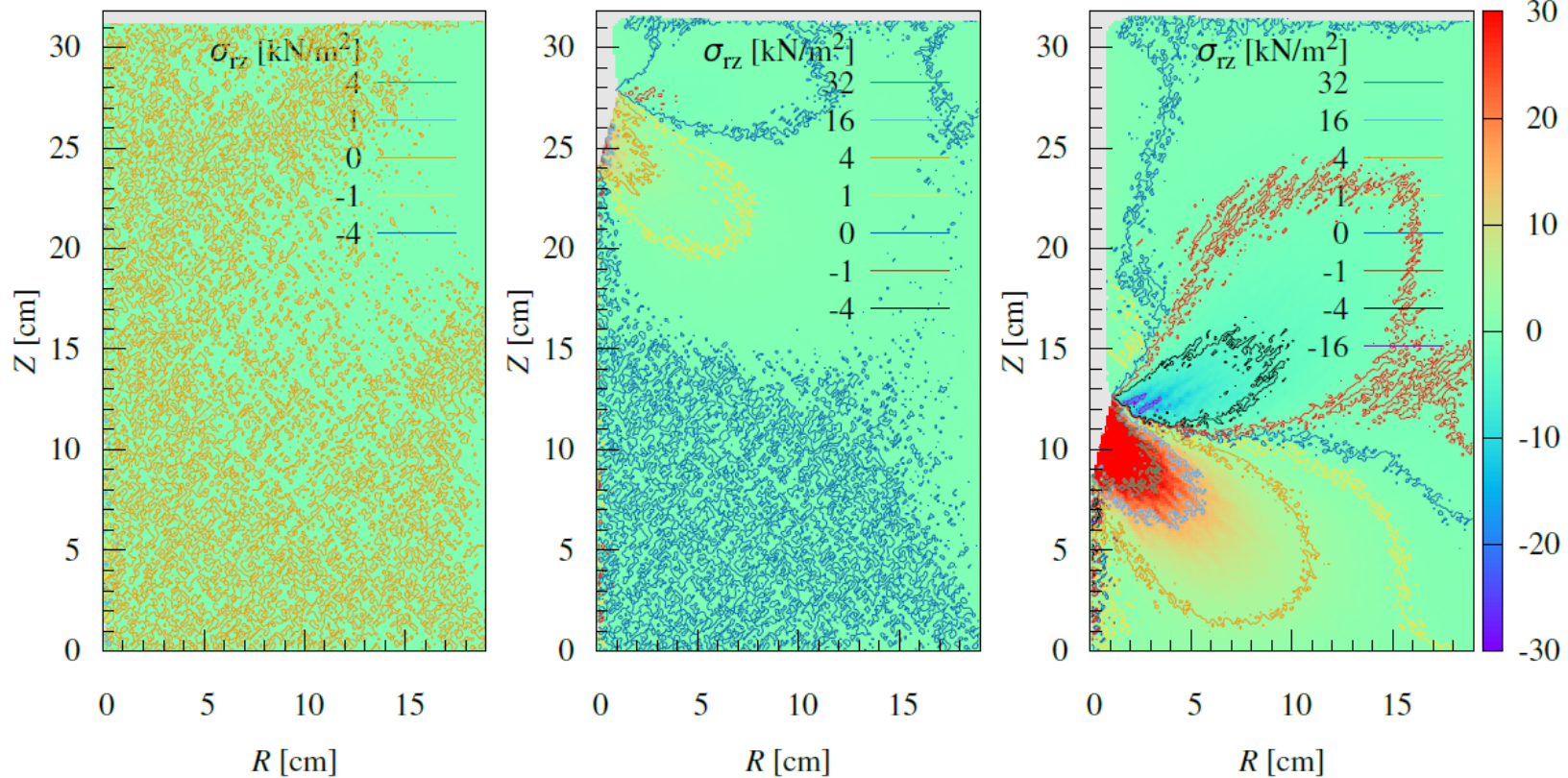
## Dense sample

### $\sigma_{rz}$ component

0.05 s

2.4 s

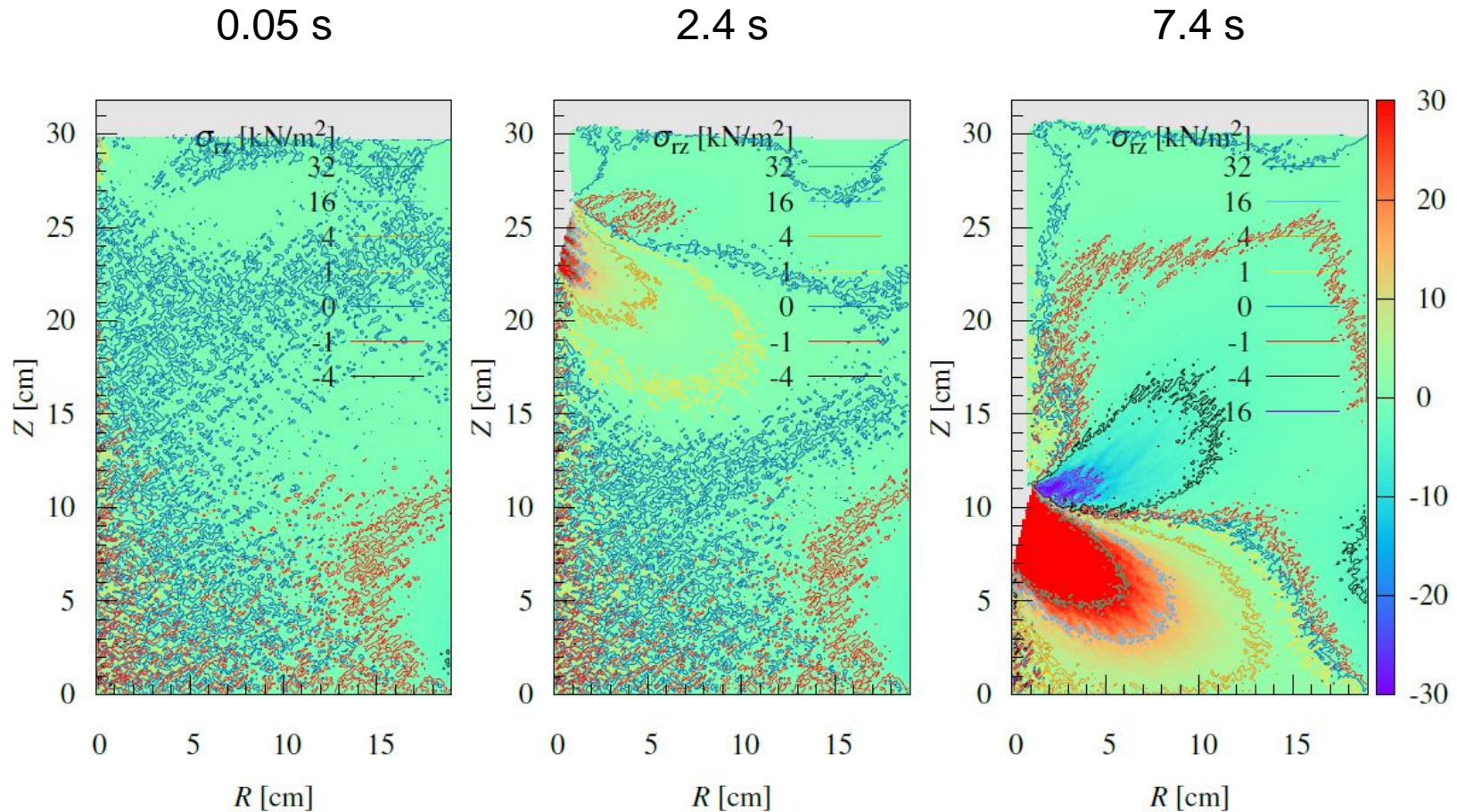
7.4 s



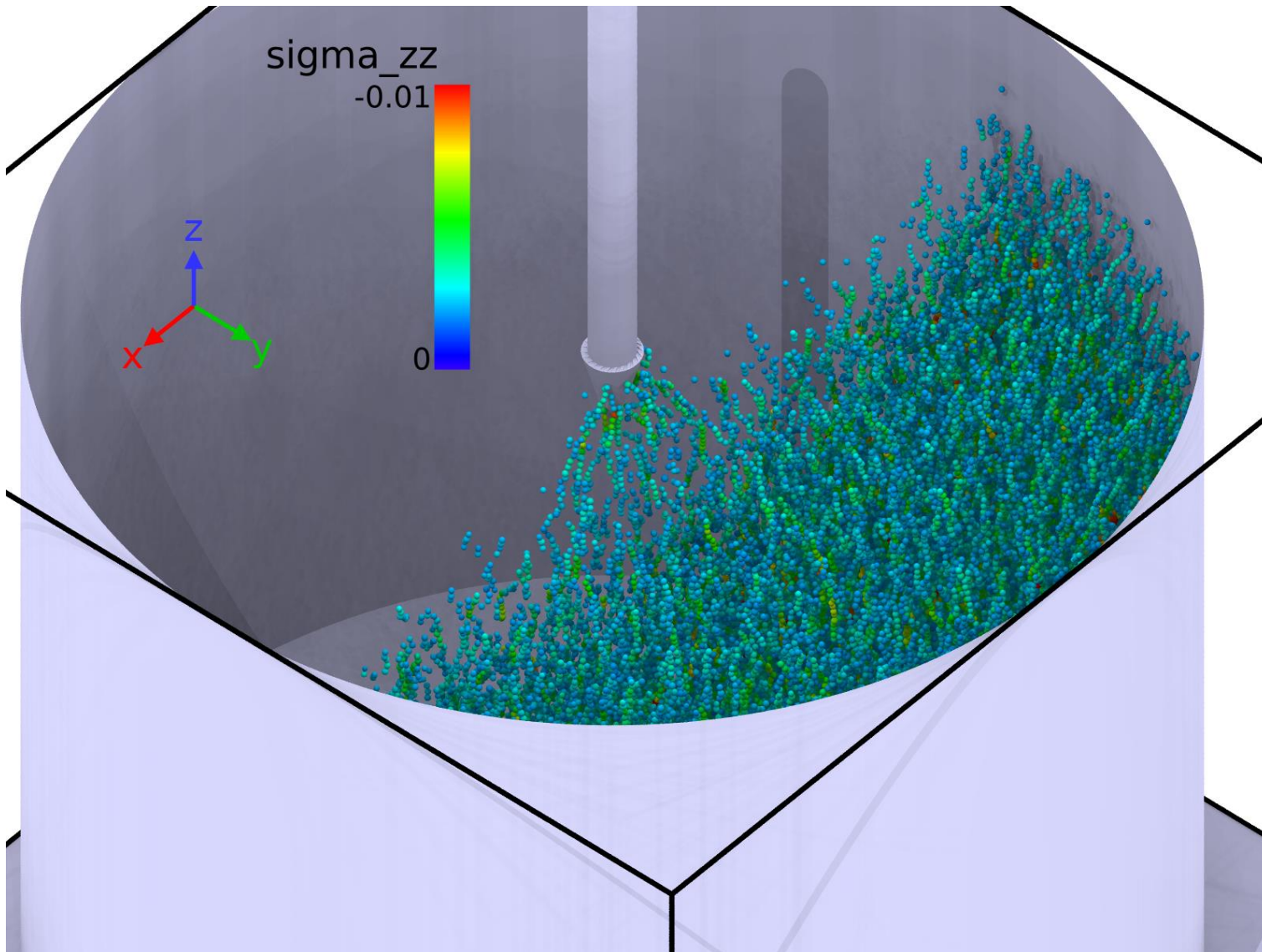
# Evaluating vertical stress during CPT

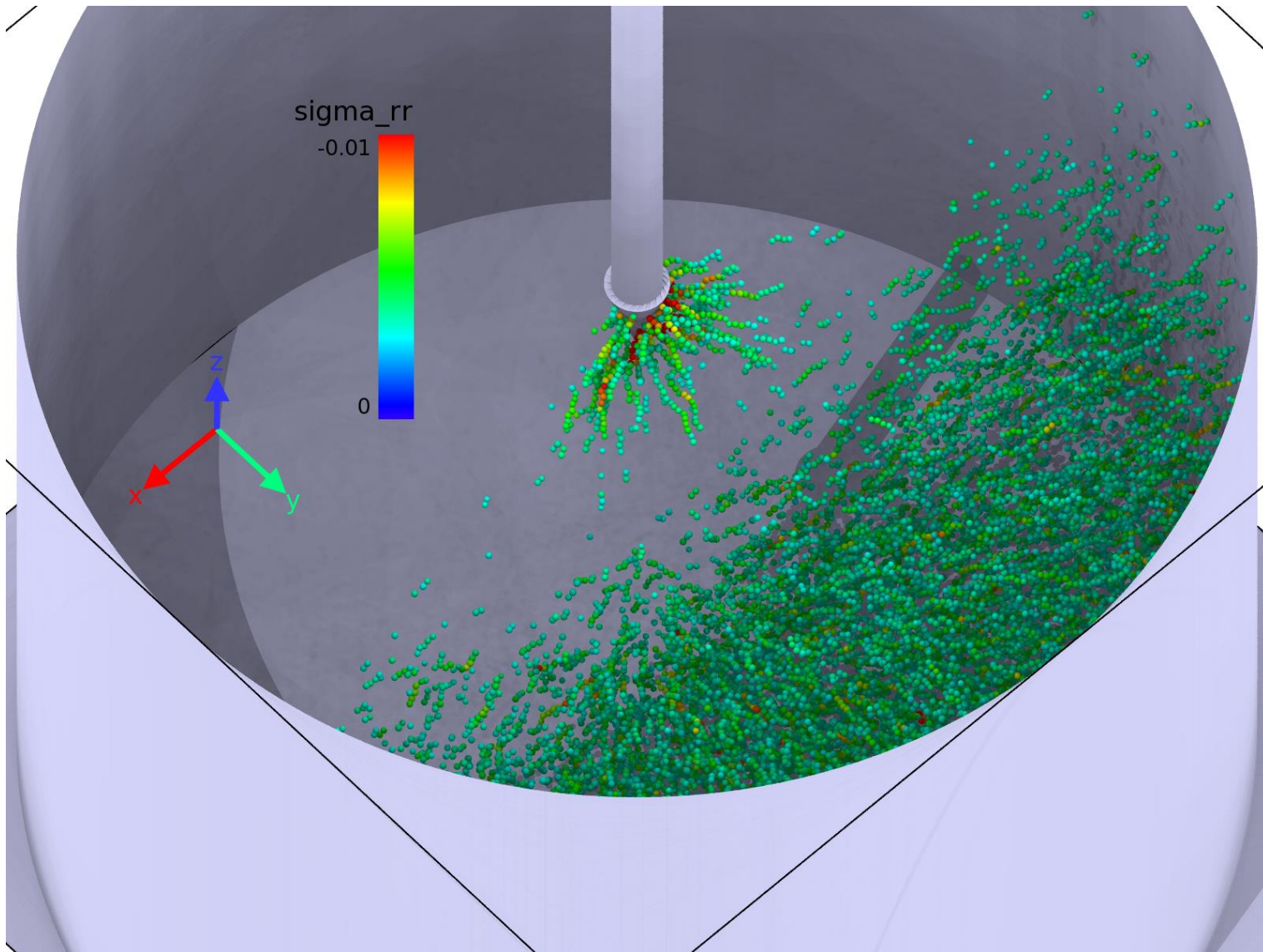
## Very dense sample

### $\sigma_{rz}$ component



# Force chains - $\sigma_{zz}$



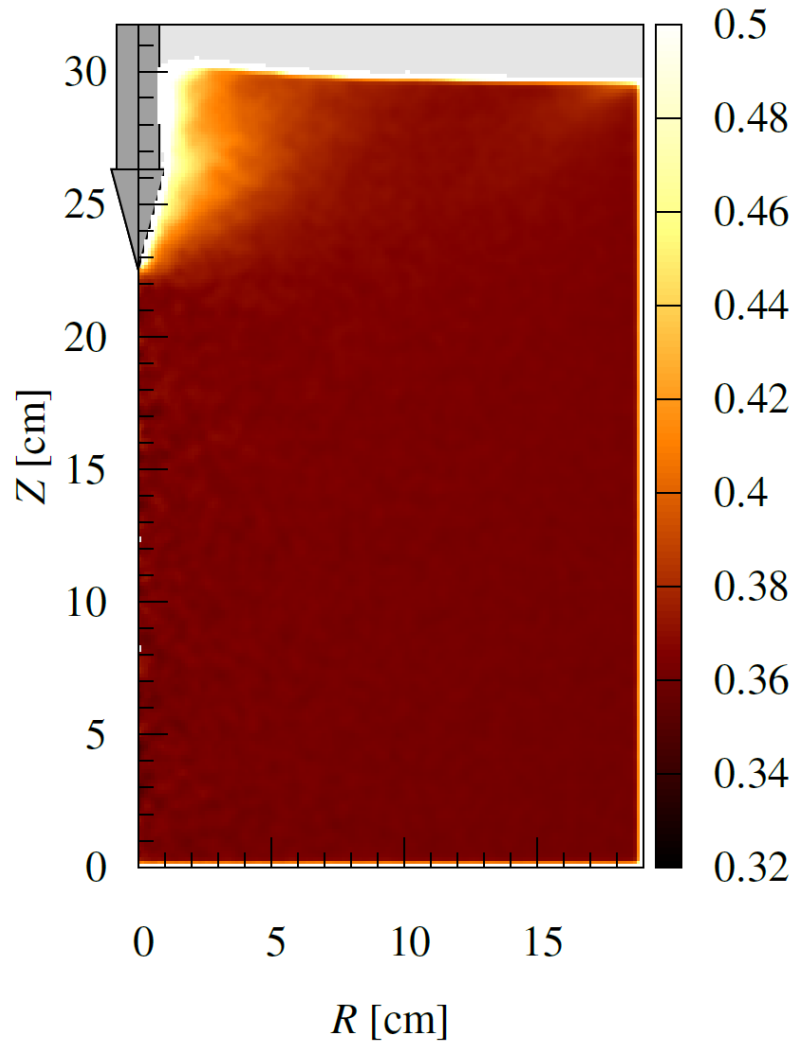
Force chains -  $\sigma_{rr}$ 



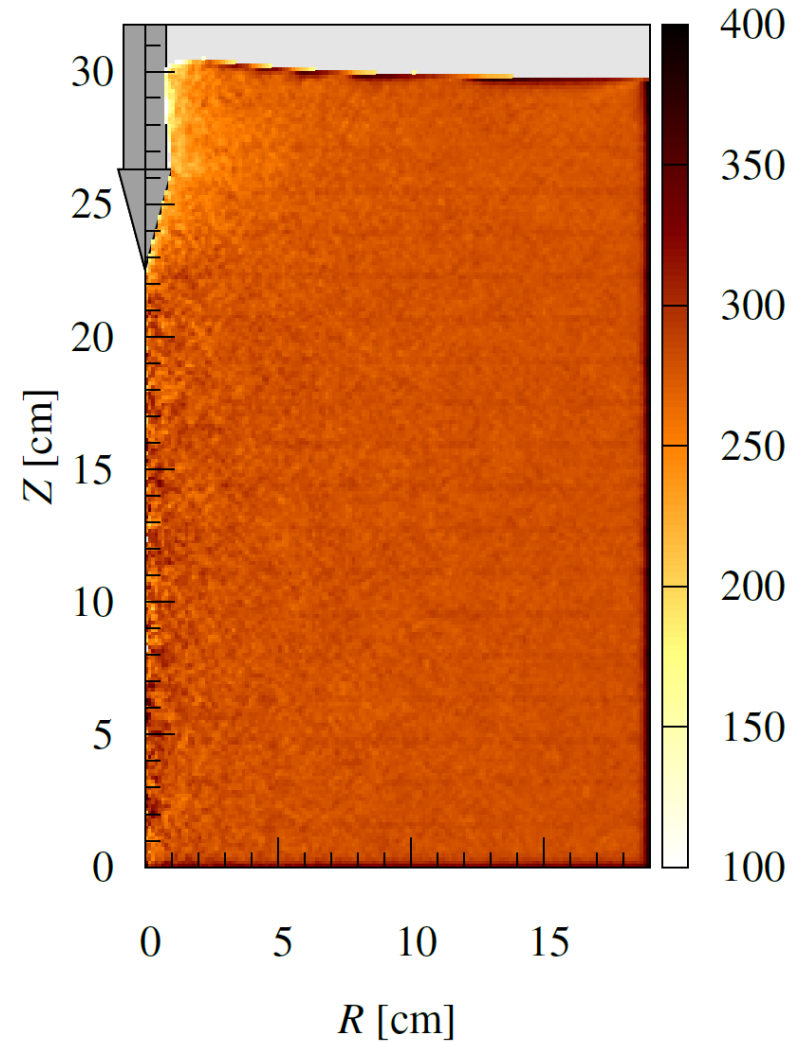
# Local particle density

Very dense sample

Local porosity



Particle number density [ $\text{cm}^{-3}$ ]



# DEM simulations steps

## Cone/wheel-subgrade interaction

Construct box and wheel of triangular facets

Settle particles in box

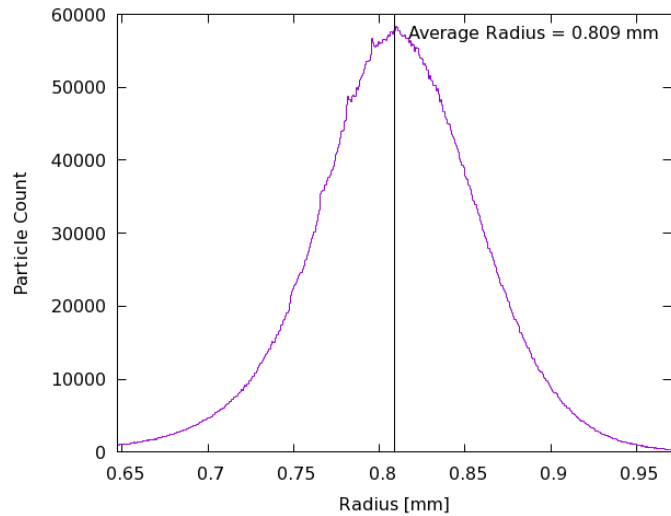
Settle wheel in box with settled particles

Execute simulations constrained to laminar motion:

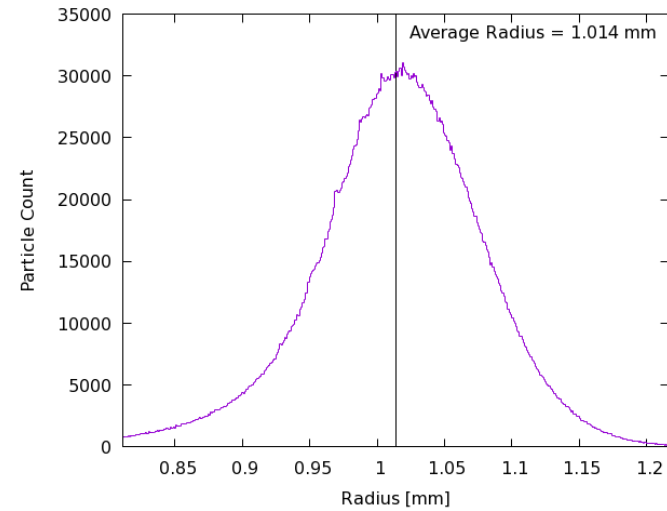
1. Apply forward velocity (towed wheel)
2. Apply torque (driven wheel)
3. Prescribe translational velocity and slip, measure traction and pull (multiple slips = pull-slip curve)

# DEM convergence study - particle size distributions

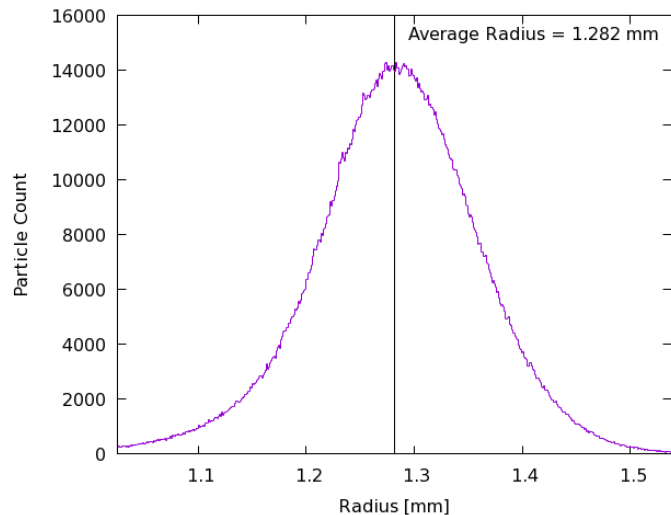
Particle size distribution, finest particles



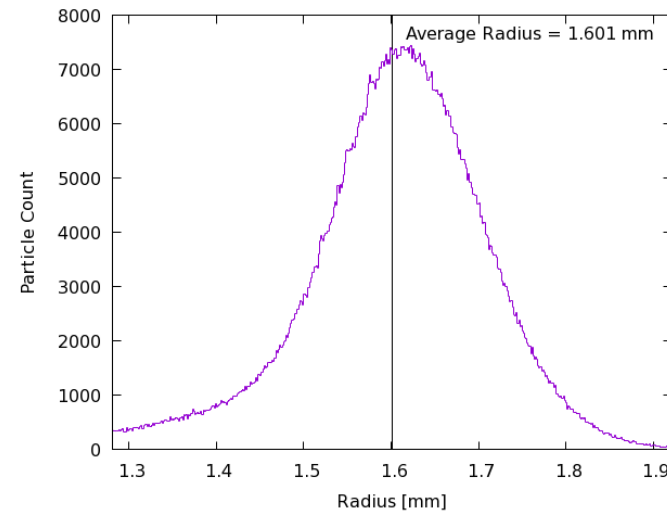
~2x lower number of particles



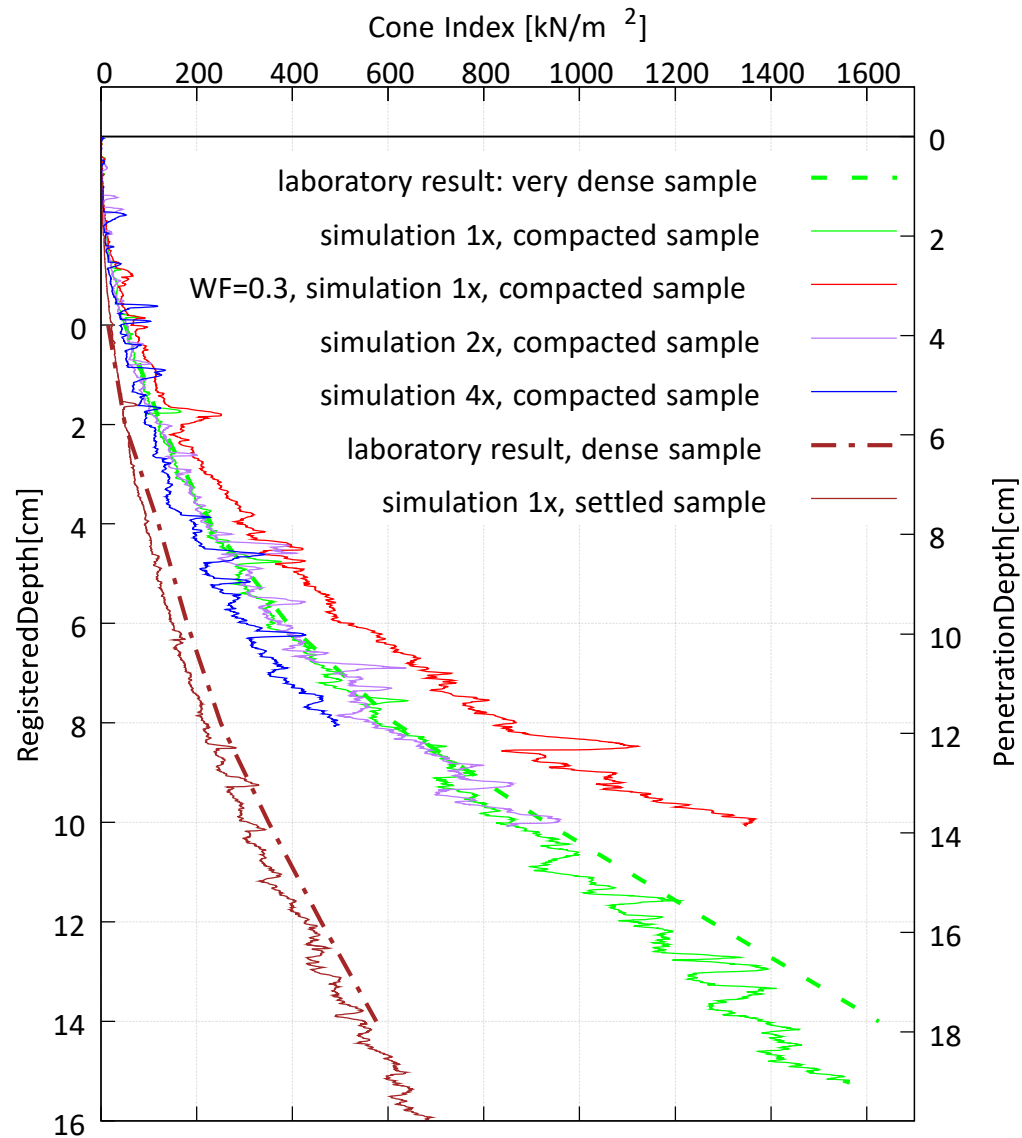
~4x lower number of particles



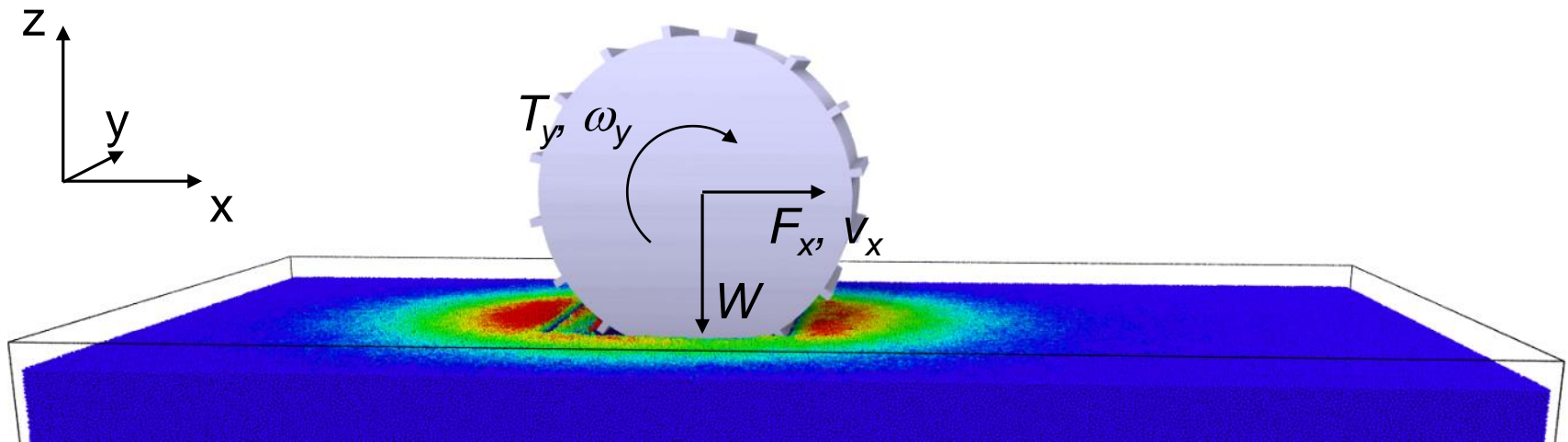
~8x lower number of particles



# DEM convergence study



# DEM model of a wheel in granular media: quantities of interest



Particles' colors represent height of the subgrade surface

Slip (powered wheel):

$$i = \frac{R\omega_y - v_x}{R\omega_y}$$

Skid (braked/towed wheel):

$$j = \frac{v_x - R\omega_y}{R\omega_y}$$

Drawbar pull force  $F_x$  (powered wheel):  
force available to pull external load

Motion resistance:  
resistance from subgrade + internal

Traction coefficient:

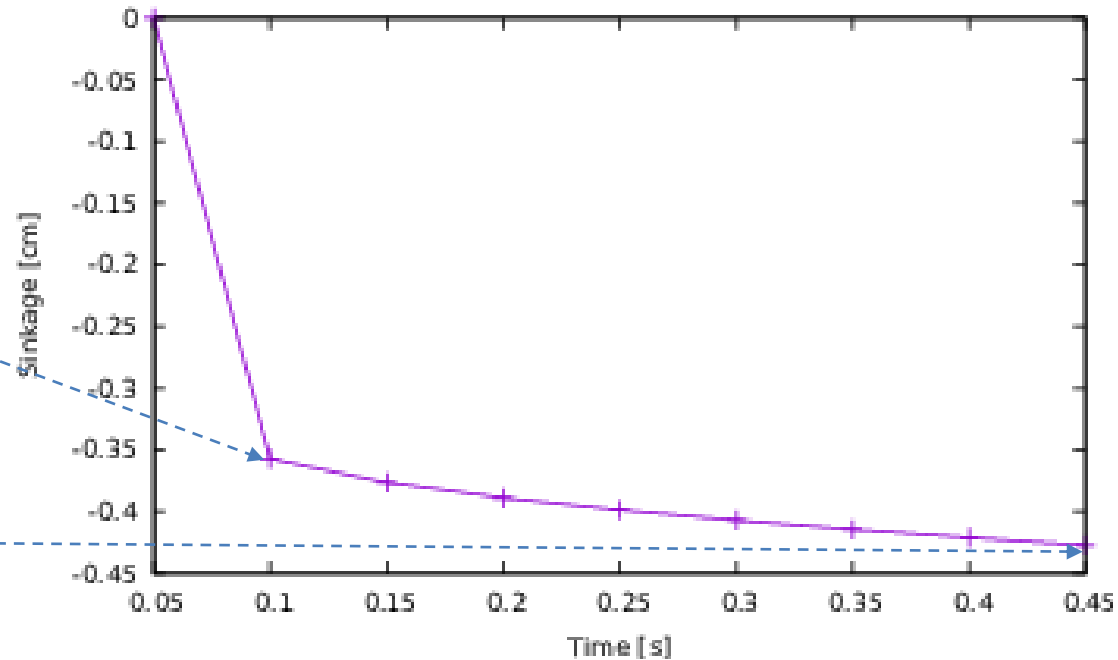
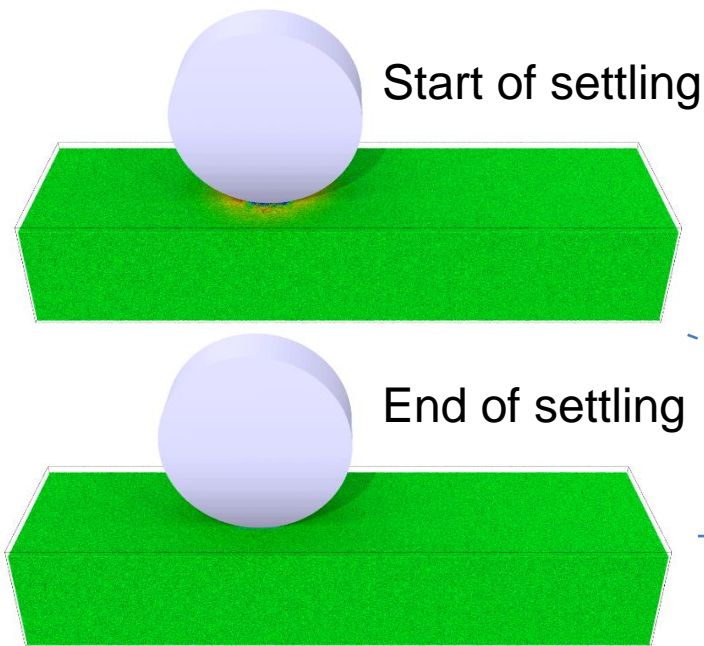
$$k = \frac{T_y}{RW}$$

Sinkage:

depth to which wheel sinks into a subgrade

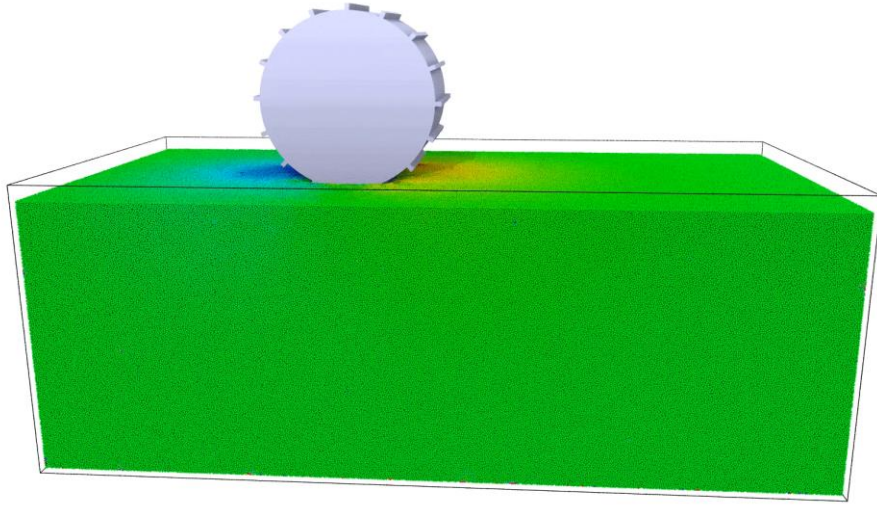
# Wheel settling

- 1) Initial location of wheel is set to just touch subgrade surface.
- 2) Wheel is constrained to move in vertical direction only.
- 3) Gravity is applied.

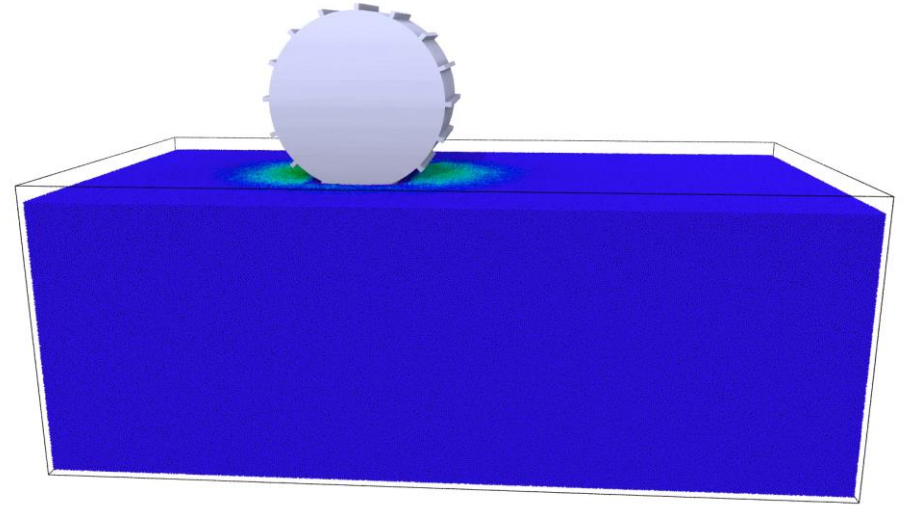


Wheel settling under gravity, after being placed on the subgrade surface. Left figures: particles' color depicts z-component of particle velocities: red is upward, blue is downward, green is zero. Right plot: sinkage of the wheel with time.

# 4" wheel driven by torque 100 lbf·ft

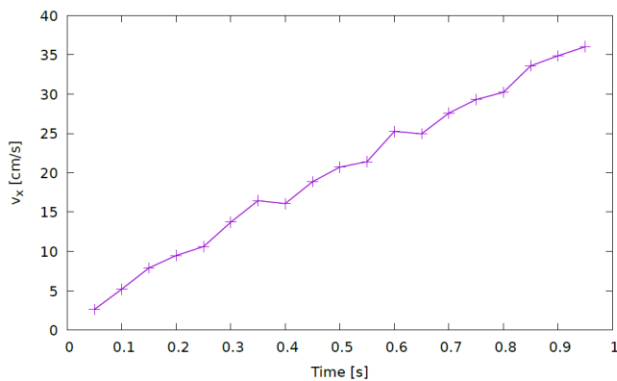


Particles' color depicts x-component of particle velocity, red is forward, blue backward, green zero.

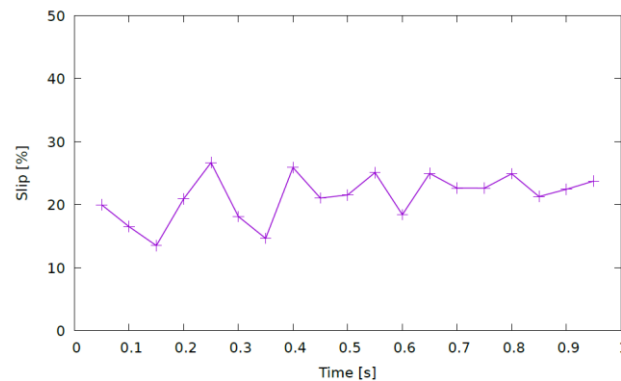


Color depicts particle height above the subgrade surface.

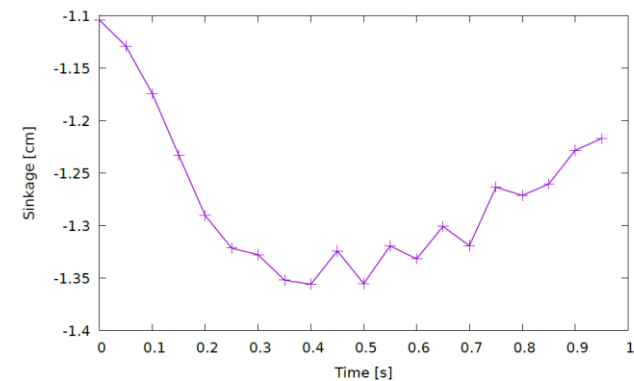
## Forward velocity



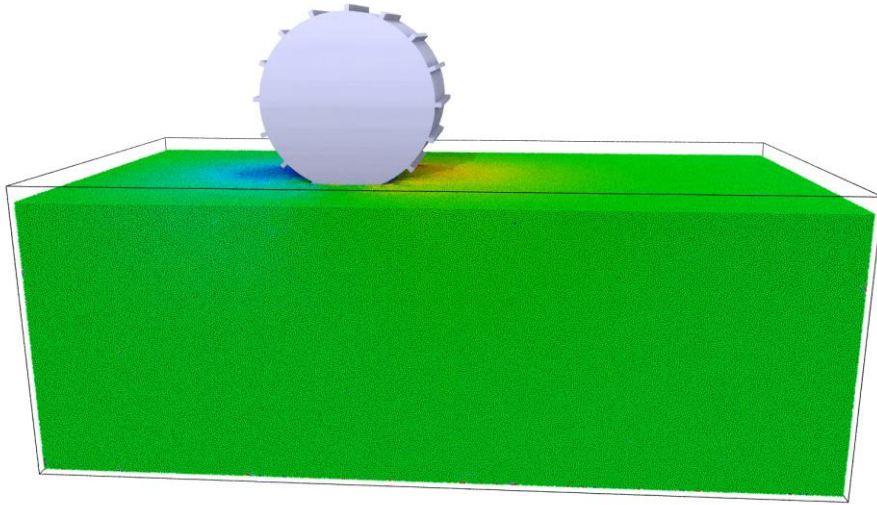
## Slip



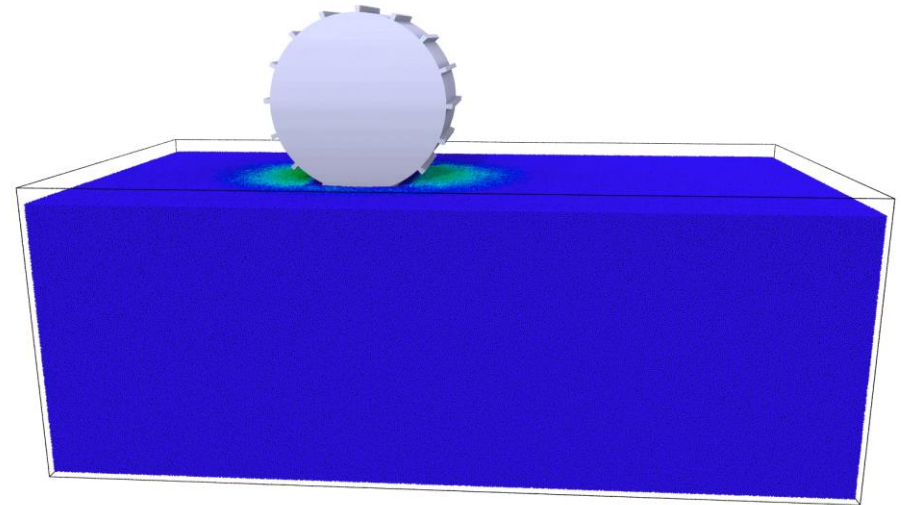
## Sinkage



# 4" wheel driven by torque 30 lbf·ft

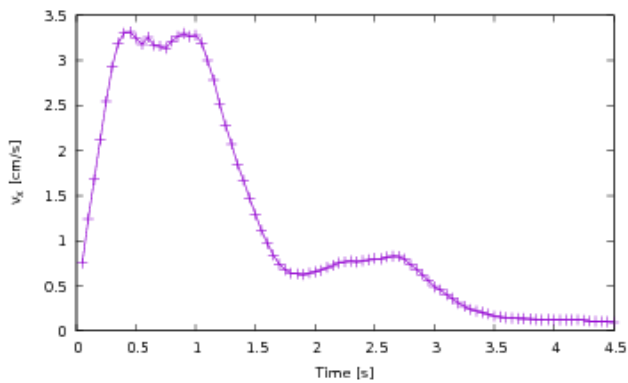


Particles' color depicts x-component of particle velocity, red is forward, blue backward, green zero.

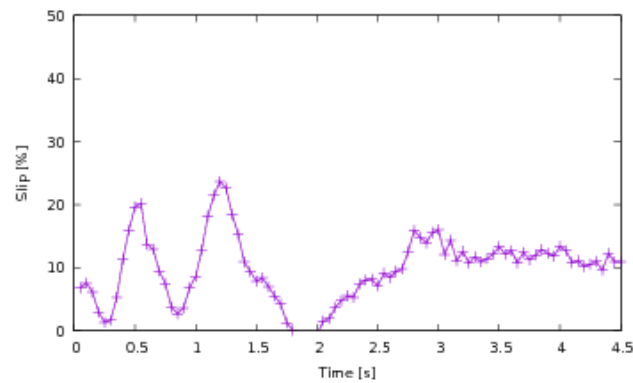


Color depicts particle height above the subgrade surface.

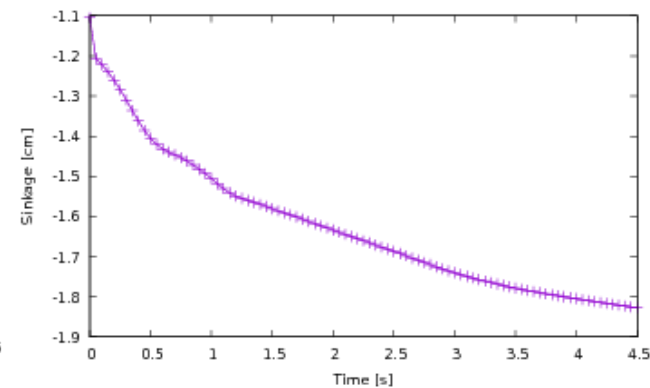
## Forward velocity



## Slip



## Sinkage



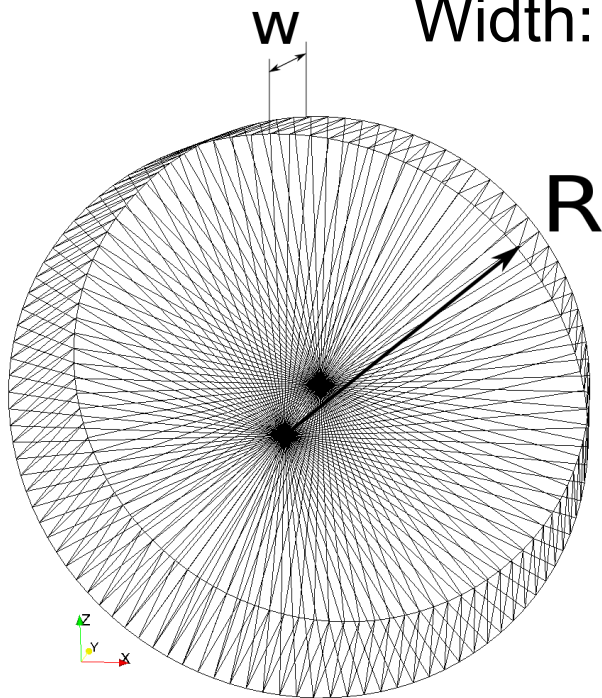


# 14.04" wheel

Smallest wheel in the DROVE database

Diameter: 14.04"

Width: 4.15"



Carriage velocity: 66 in/s

Wheel rotation velocity: 82.5 in/s

Cone index: 416 kPa

Weight: 42 lbf

Sinkage / tire diameter: 1.4 %

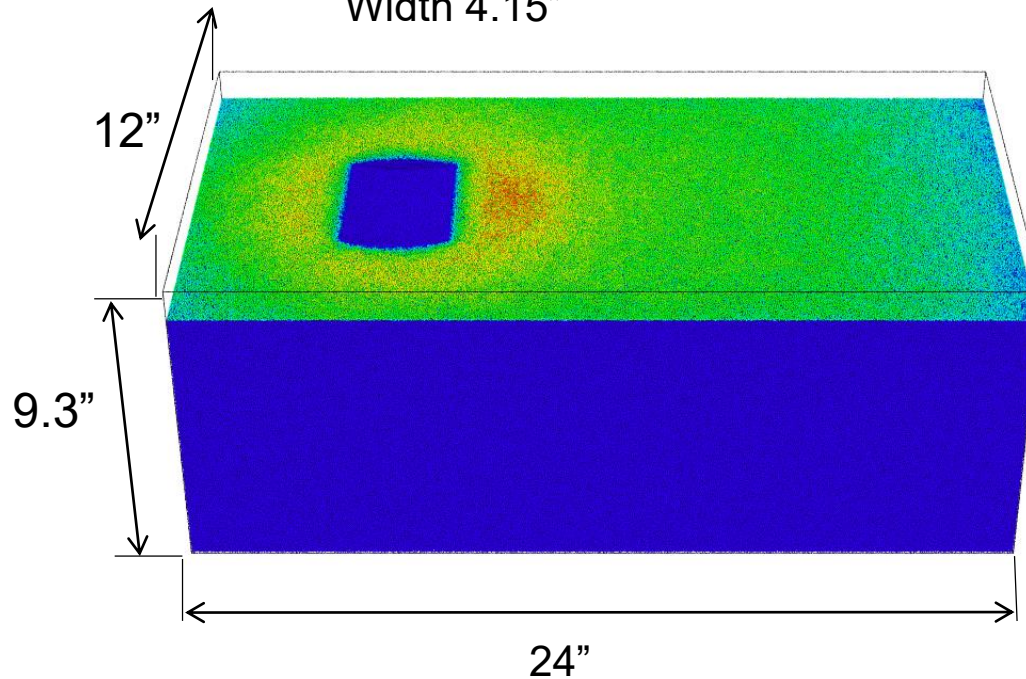
Torque: 13 lbf·ft

DBP: 40.5 %

# DEM simulations for slip-pull curve: dimensions

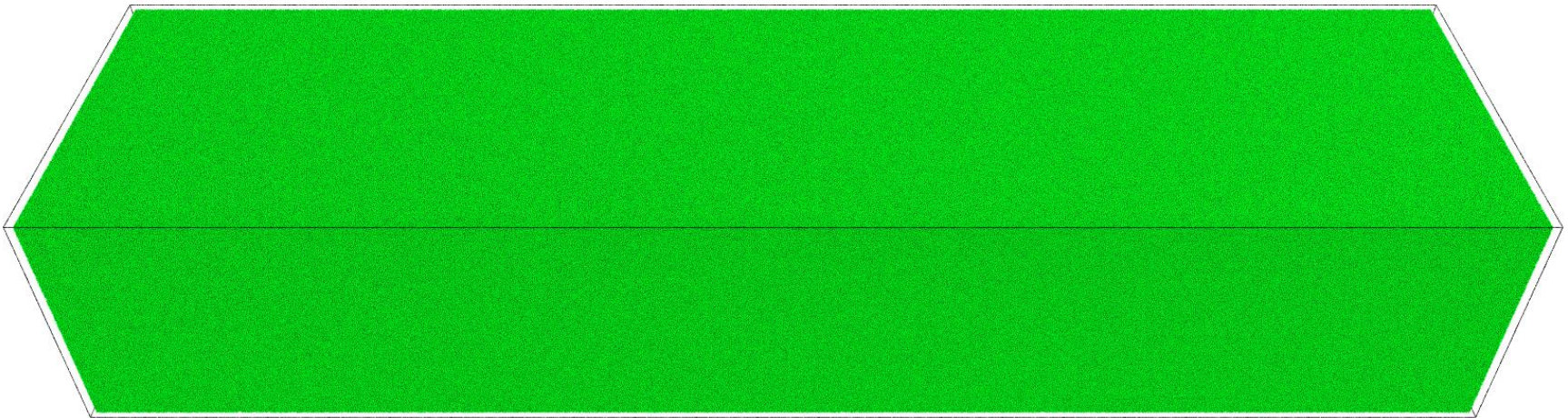
Smallest wheel in the DROVE database

Wheel:  
Diameter 14.04"  
Width 4.15"



# Preparation of larger particle bed for realistic size wheel

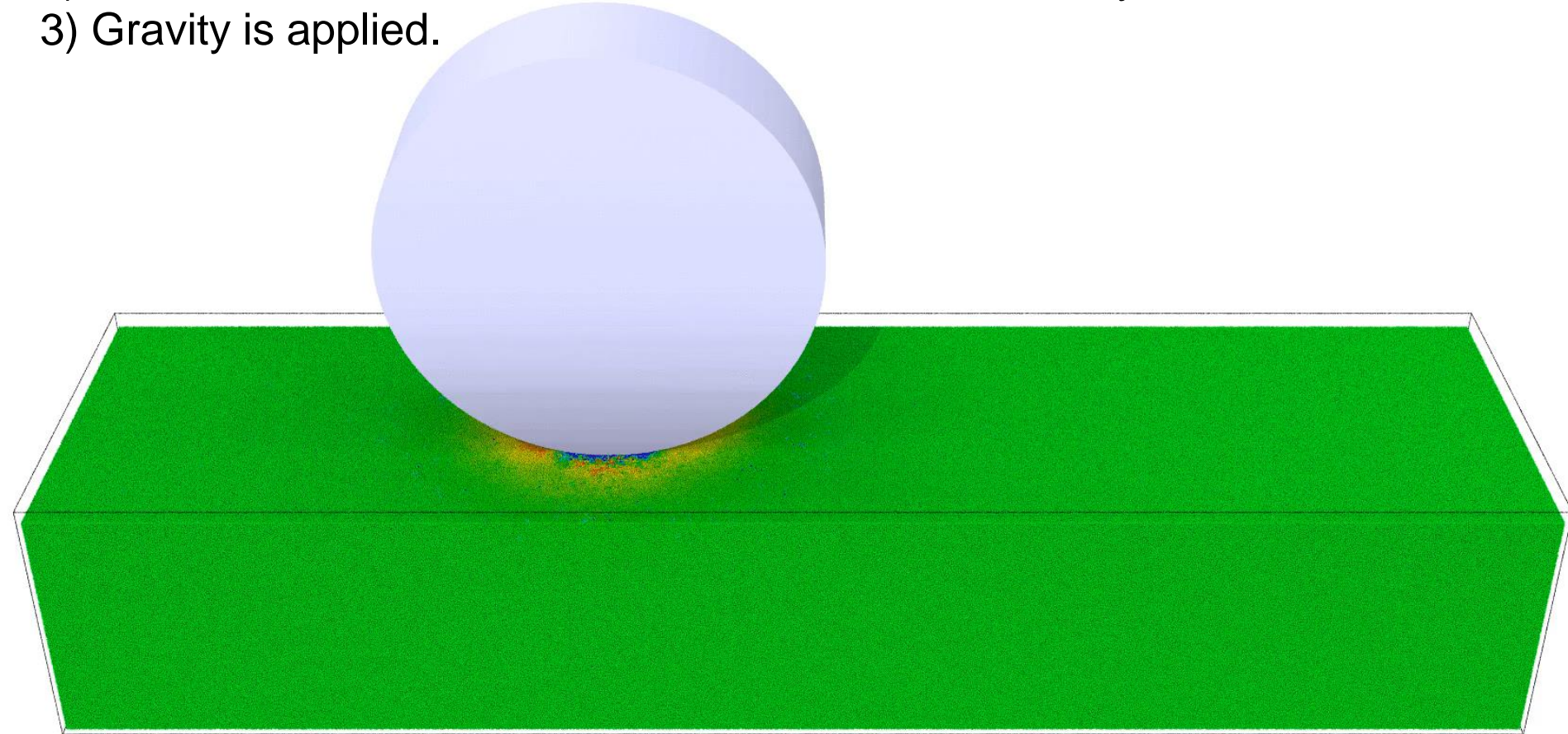
- 1) Particles are placed in space by filling tetrahedral mesh created by TetGen mesh generation package.
- 2) Particles are settled under gravity. Settling is complete when average kinetic energy decreases to chosen threshold.



Particle settling under gravity - after initial random placement within a box.  
Particles' color depicts z-component of the particle velocity, red is upward, blue downward, green zero

# 14.04" wheel Settling under gravity

- 1) Initial location of wheel is set to just touch subgrade surface.
- 2) Wheel is constrained to move in vertical direction only.
- 3) Gravity is applied.



Wheel settling under gravity - after being placed on the top of particle bed. In the animation, particles' color depicts z-component of the particle velocity: red is upward, blue downward, green zero.

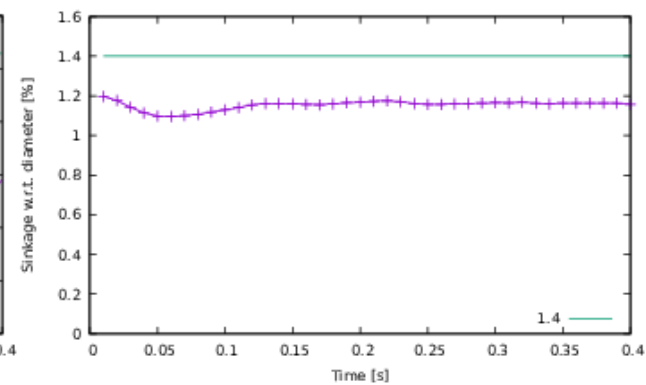
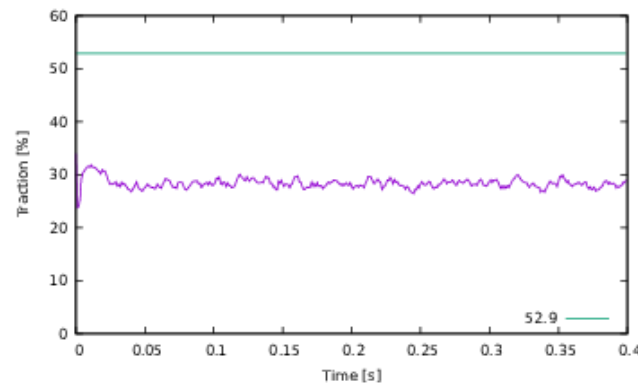
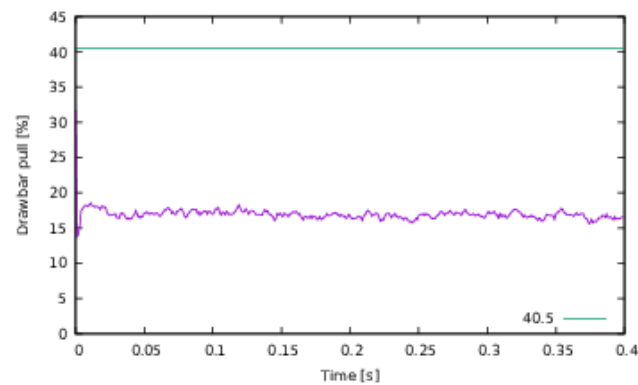
14.04" wheel  
 allow free vertical movement  
 prescribe 20% slip & carriage velocity

Color depicts x-component of particles' velocity. Red is forward, blue backward, green zero.

Drawbar pull

Traction

Sinkage



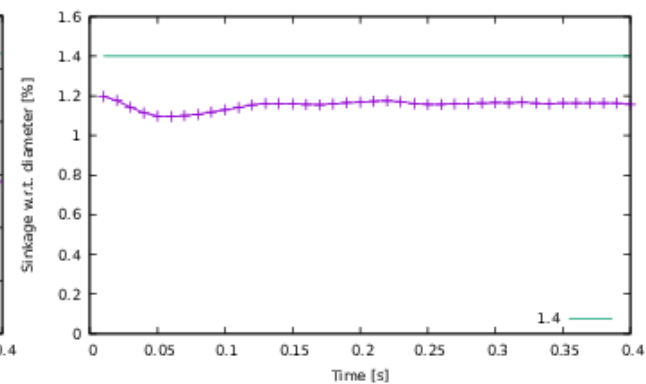
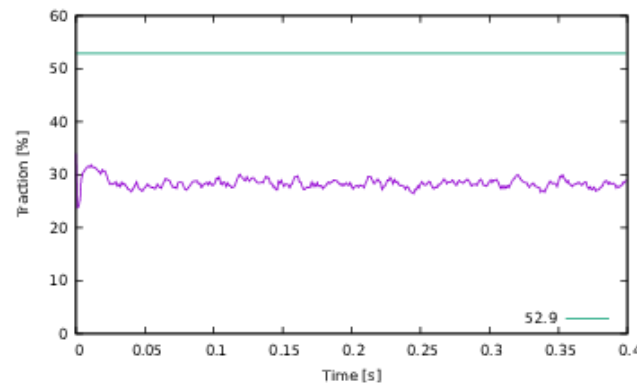
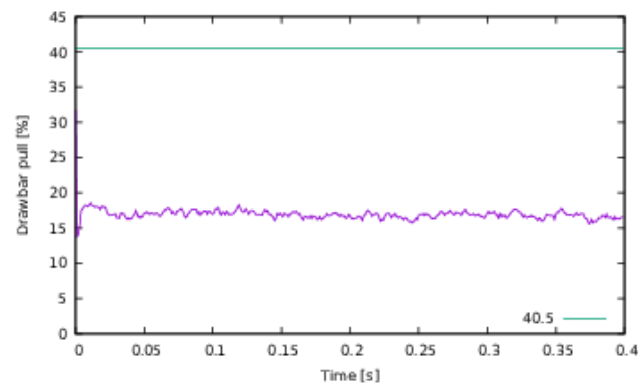
14.04" wheel  
 allow free vertical movement  
 prescribe 20% slip & carriage velocity

Color depicts z-component of particles' velocity. Red is up, blue down, green zero.

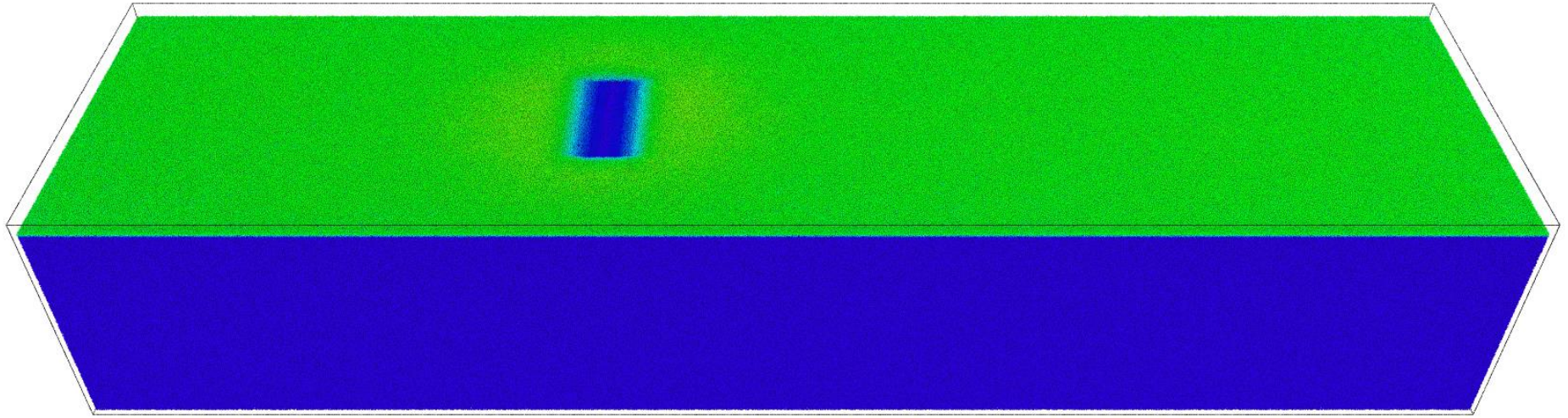
Drawbar pull

Traction

Sinkage

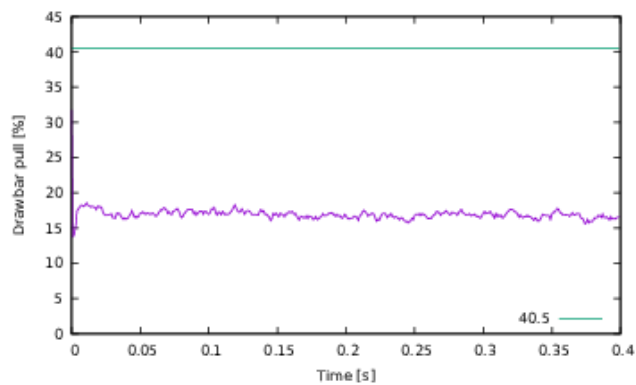


# 14.04" wheel

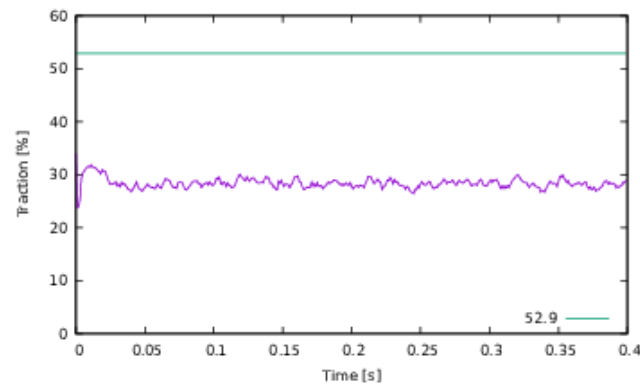


Color depicts particle height above the subgrade surface.

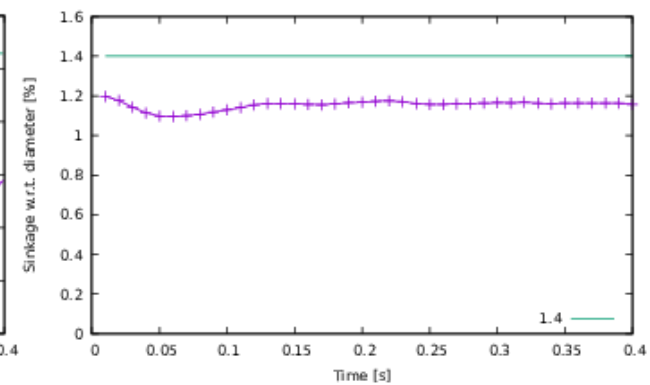
## Drawbar pull



## Traction



## Sinkage

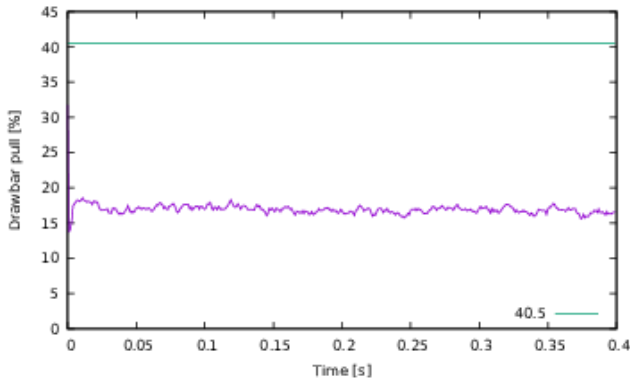


# 14.04" wheel

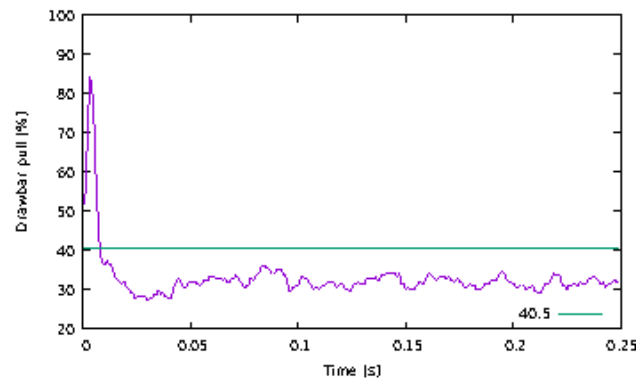
## Calibration of DEM parameters to match experimental drawbar pull and traction

### Drawbar pull:

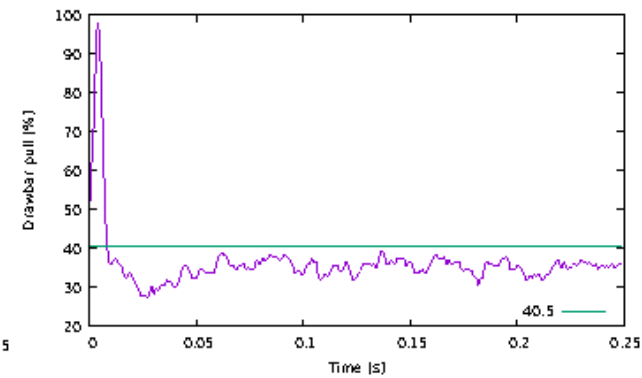
Particle-wall friction = 0.3  
rolling friction = 0.0



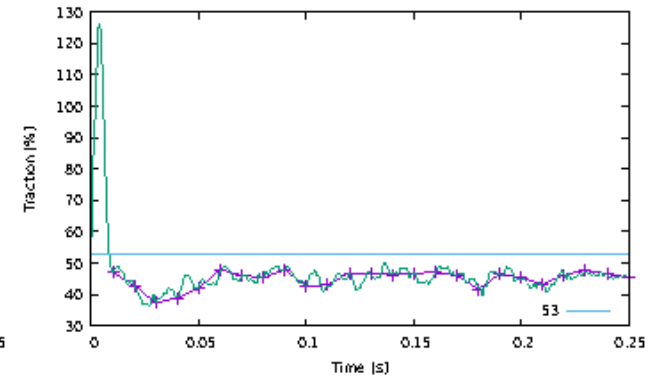
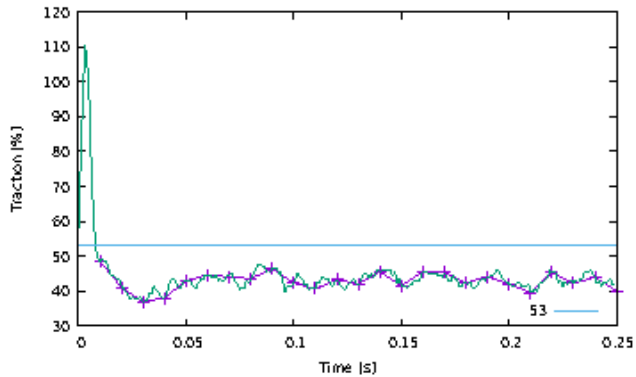
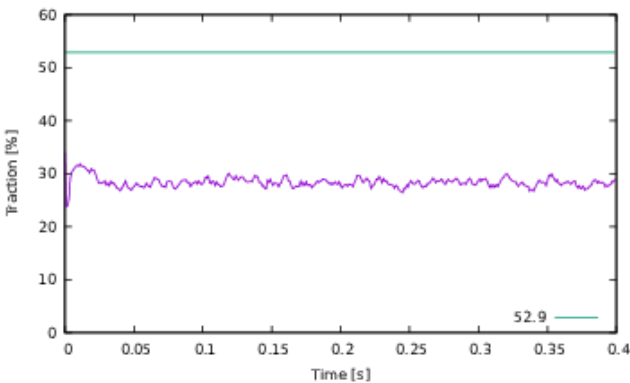
Sample density 97 lb/cu ft  
Particle-wall friction = 0.7  
rolling friction = 0.1



Particle solid density 1.5x larger  
Sample density 146 lb/cu ft



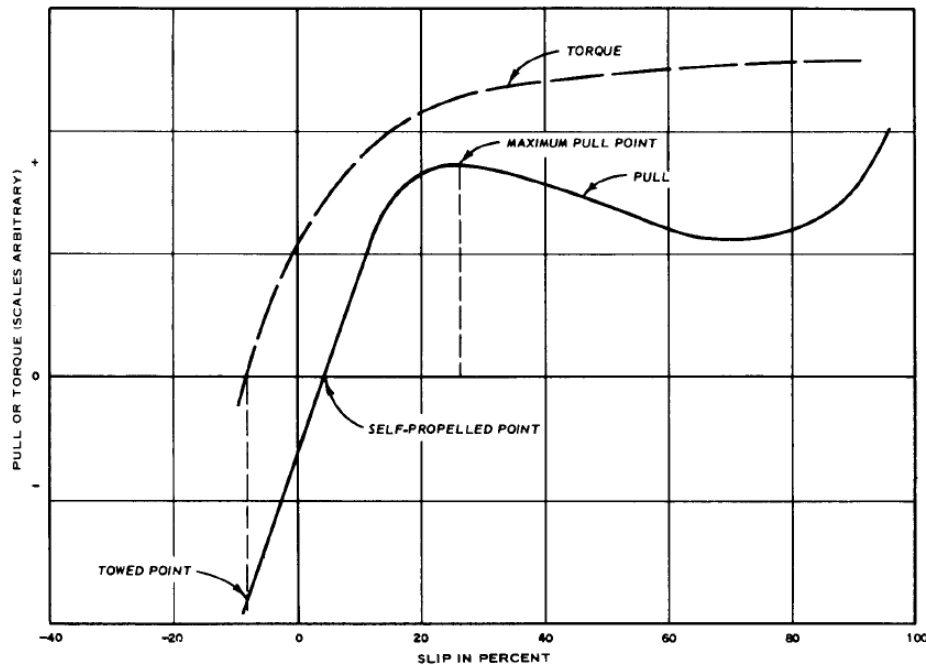
### Traction:



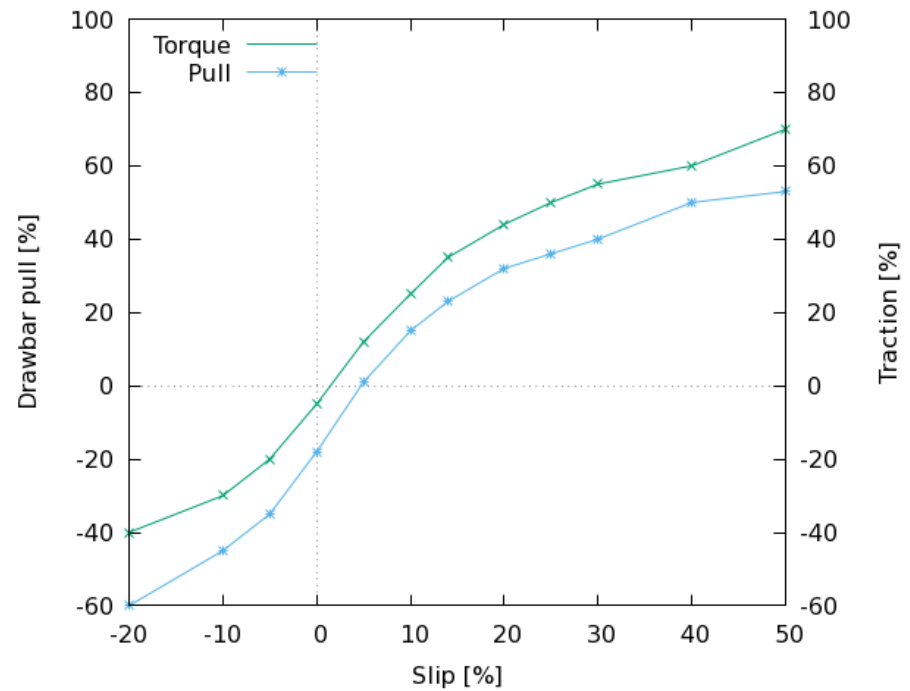


# Sample lab test pull-slip curve vs. DEM 14.04" wheel simulations

## Turnage, 1972

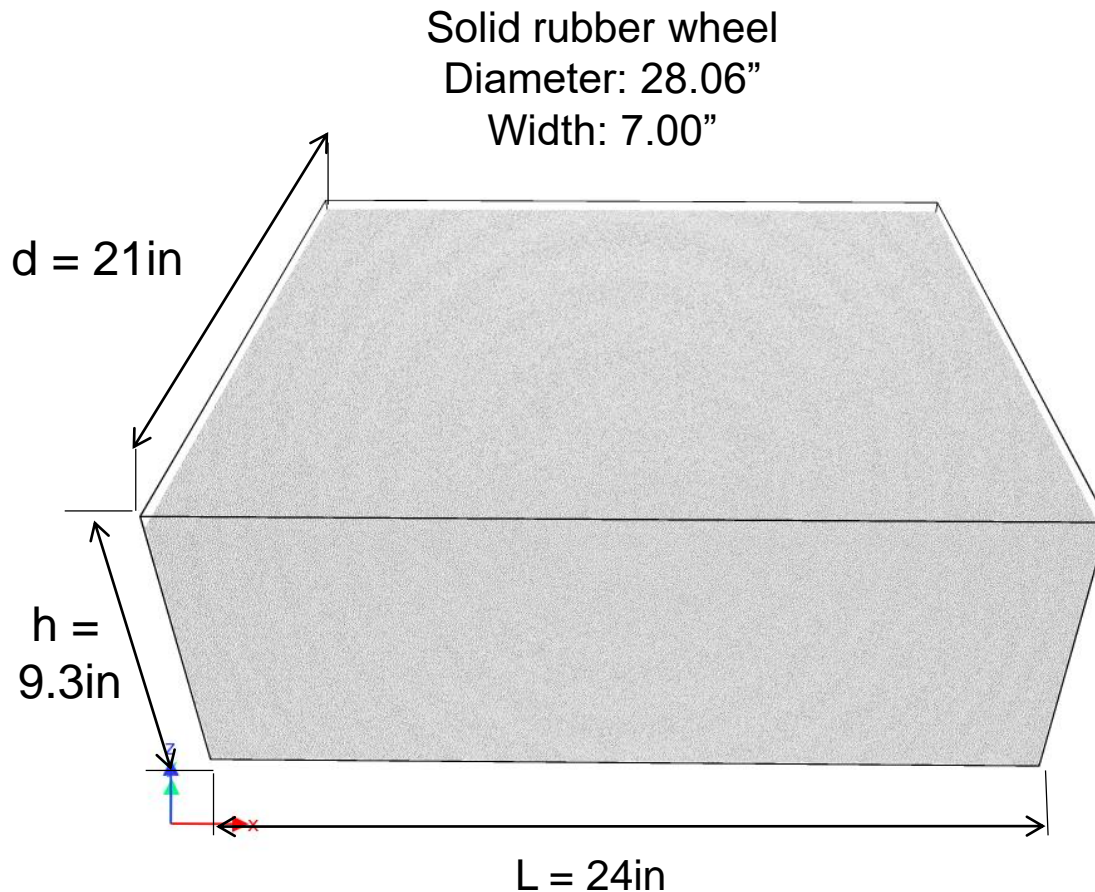


## DEM simulations

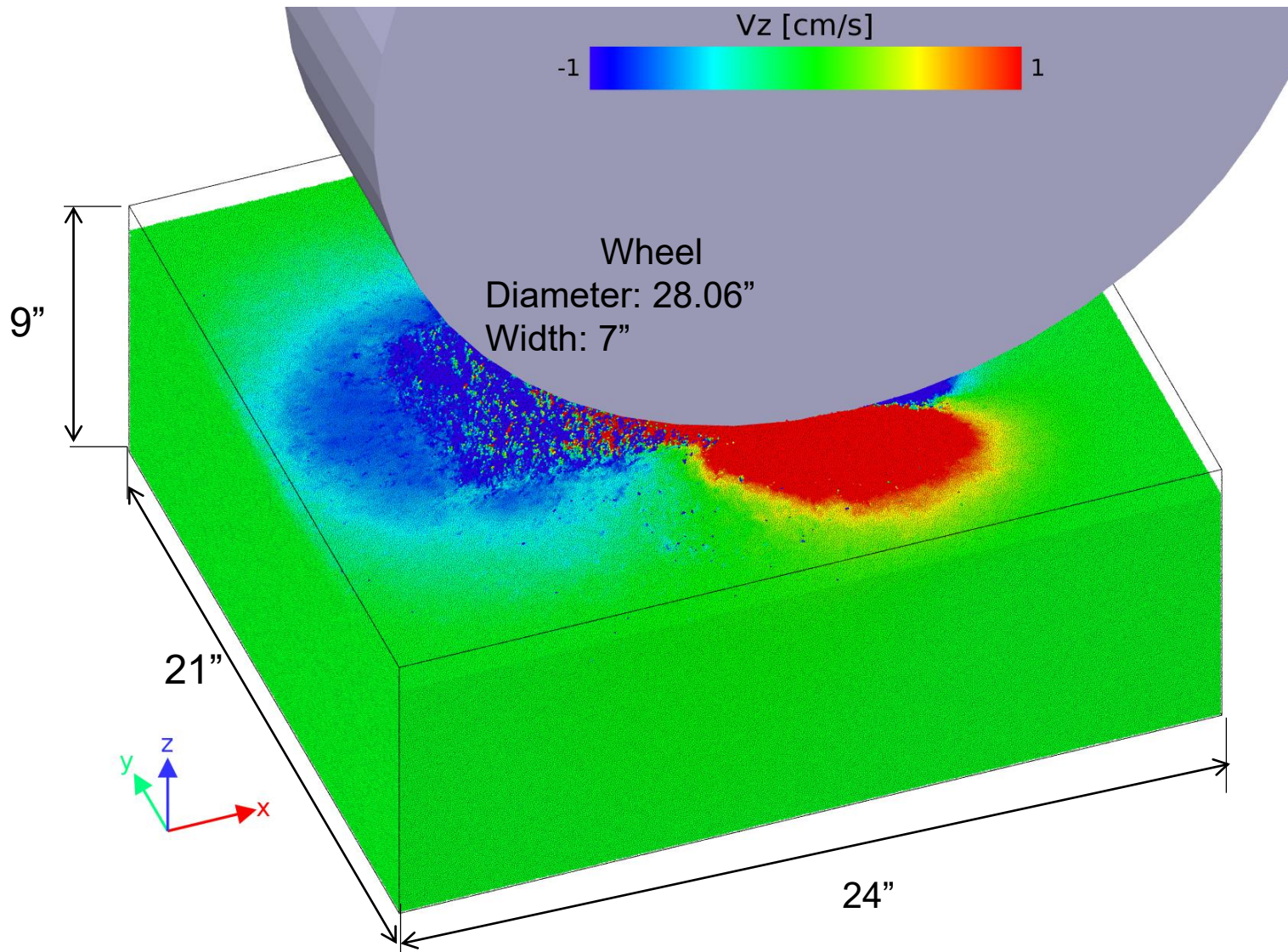


# DEM model of a wheel interaction with granular media - dimensions

Smallest low-deflection wheel from the DROVE database



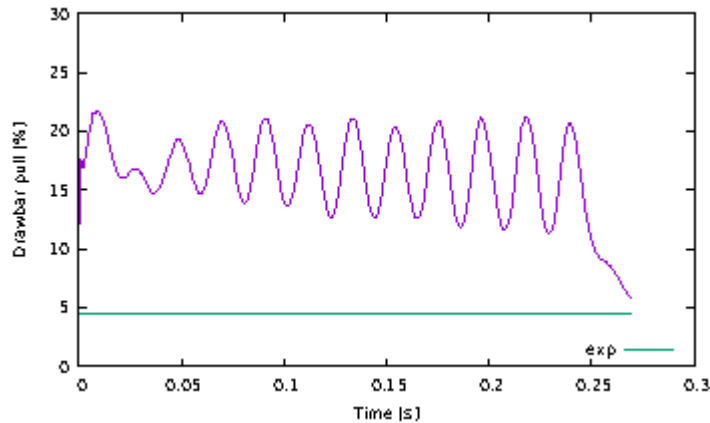
# Smallest solid rubber wheel from DROVE - coarse wheel surface facets



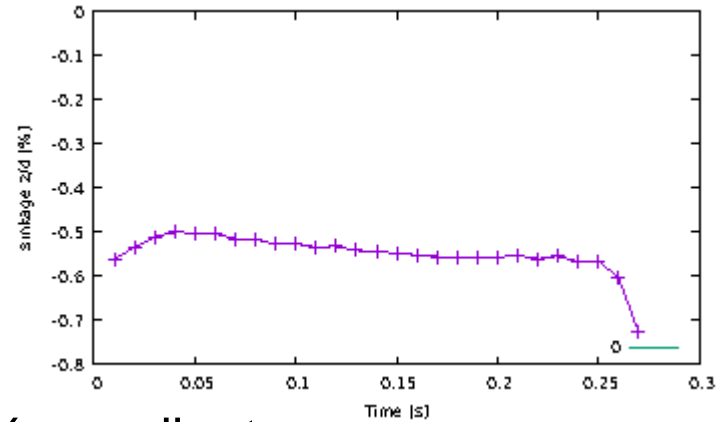
# 28.06" wheel

## smallest solid rubber wheel in DROVE database: drawbar pull, traction, and sinkage

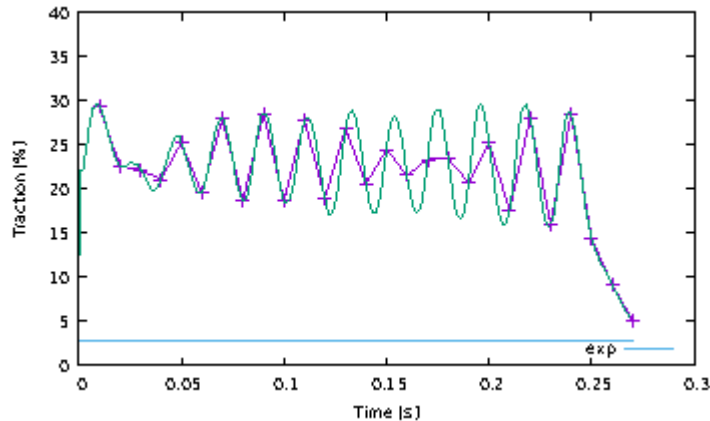
### Drawbar pull:



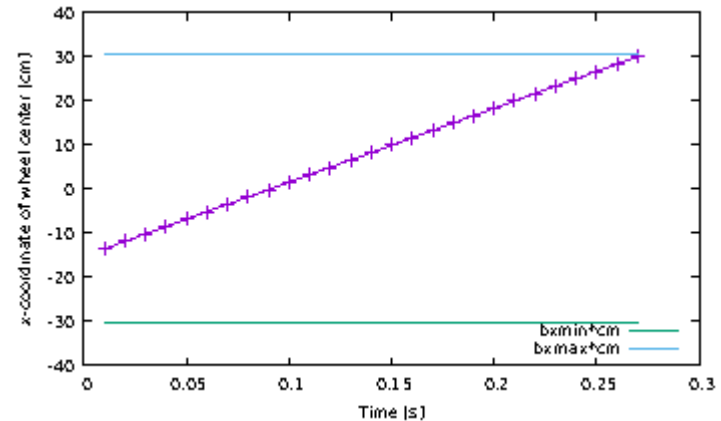
### Sinkage:



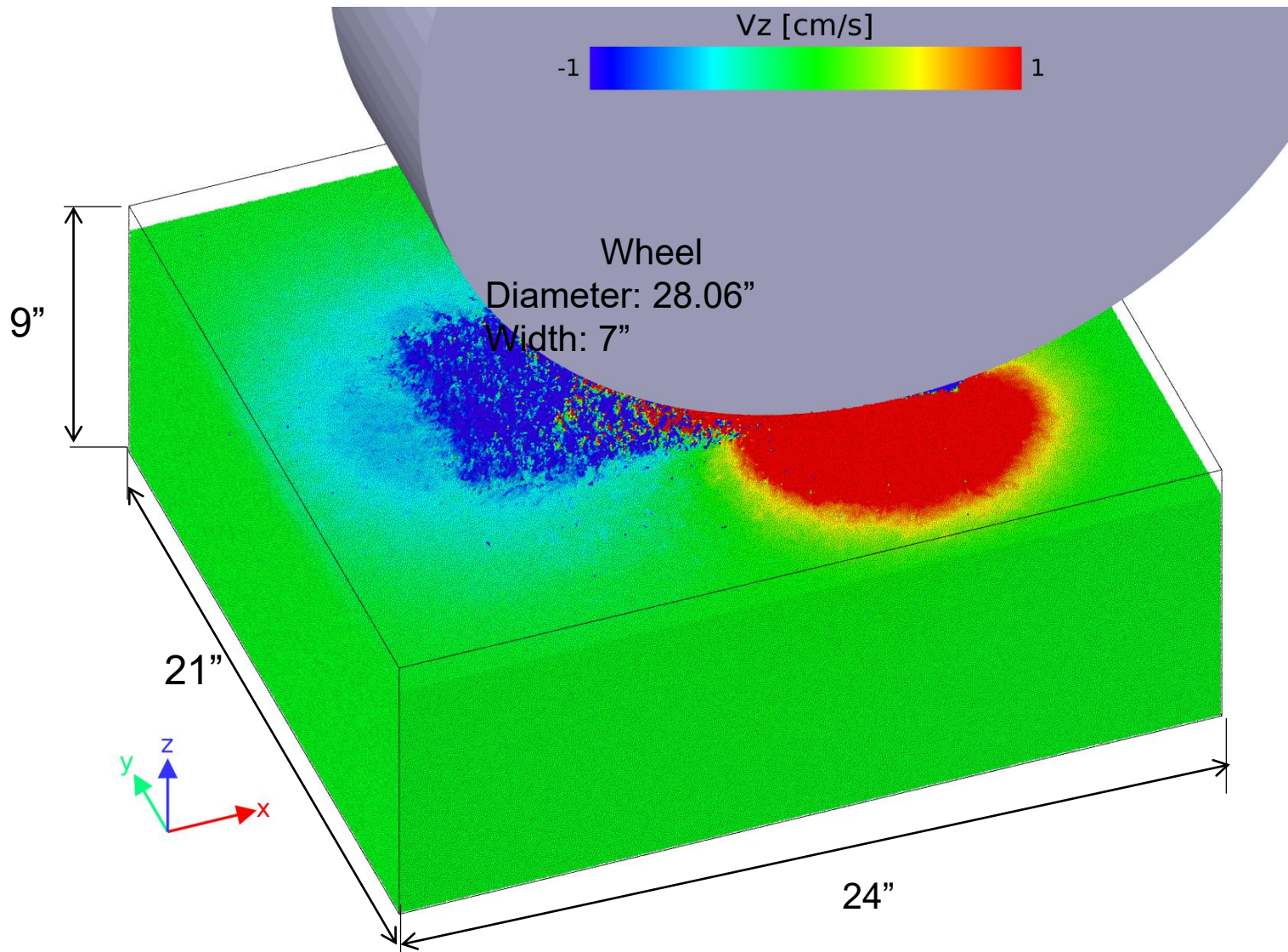
### Traction:



### X-coordinate:



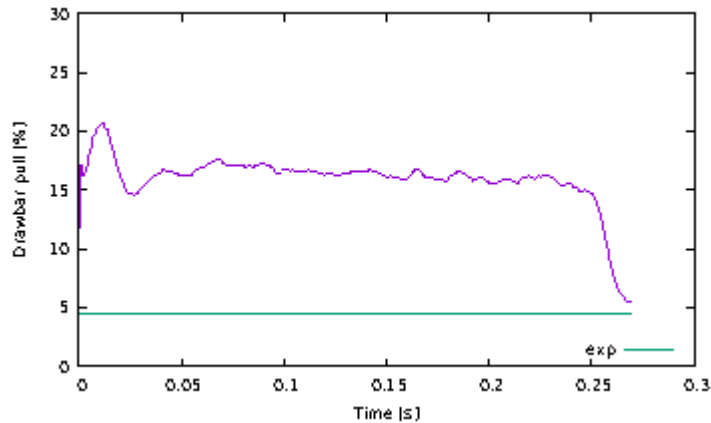
# Smallest solid rubber wheel from DROVE - refined wheel surface facets



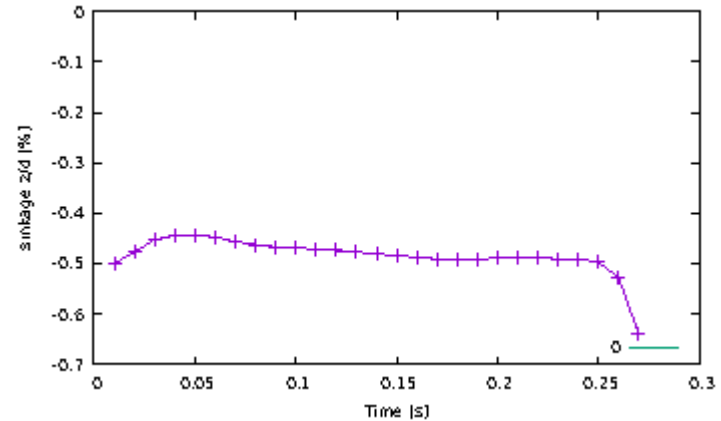
# 28.06" wheel

## smallest solid rubber wheel in DROVE database: drawbar pull, traction, and sinkage

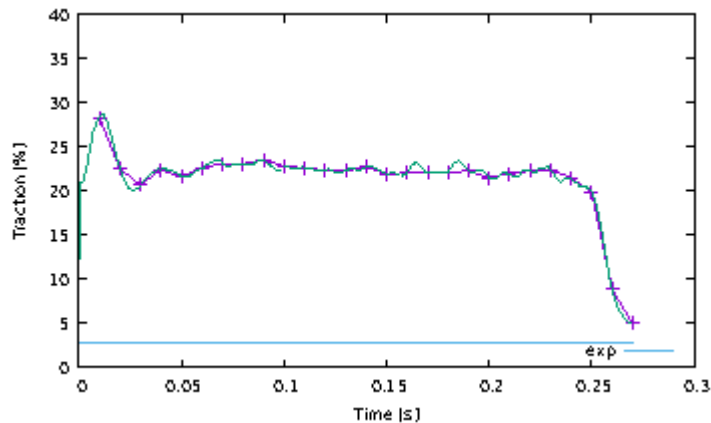
### Drawbar pull:



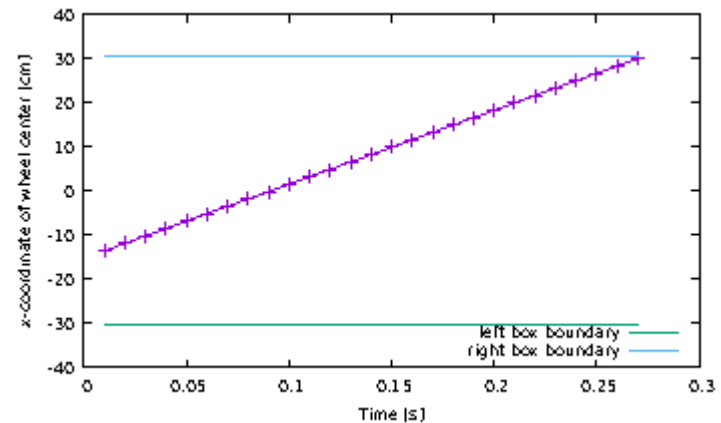
### Sinkage:



### Traction:

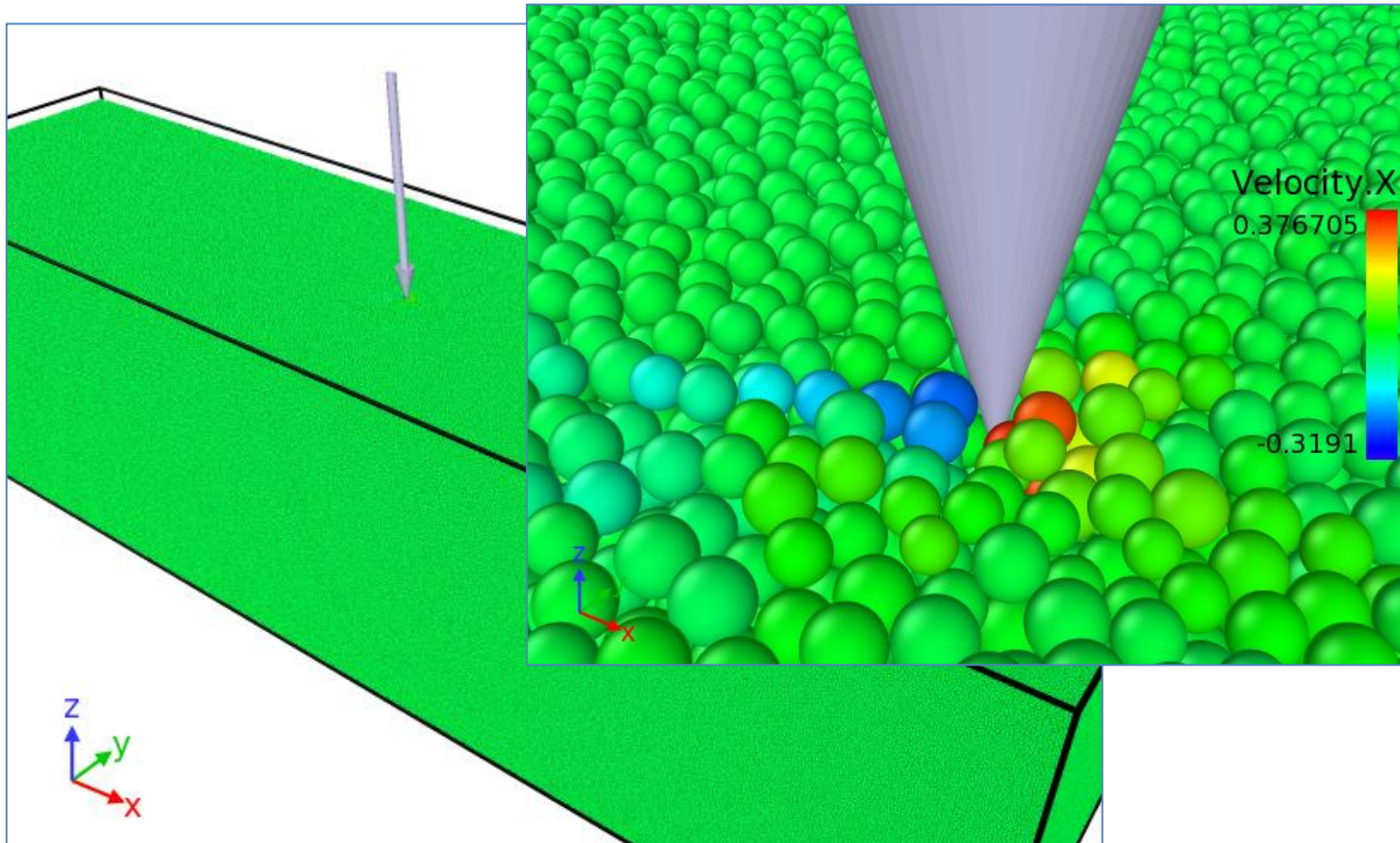


### X-coordinate:



# Cone penetrometer test large particle beds

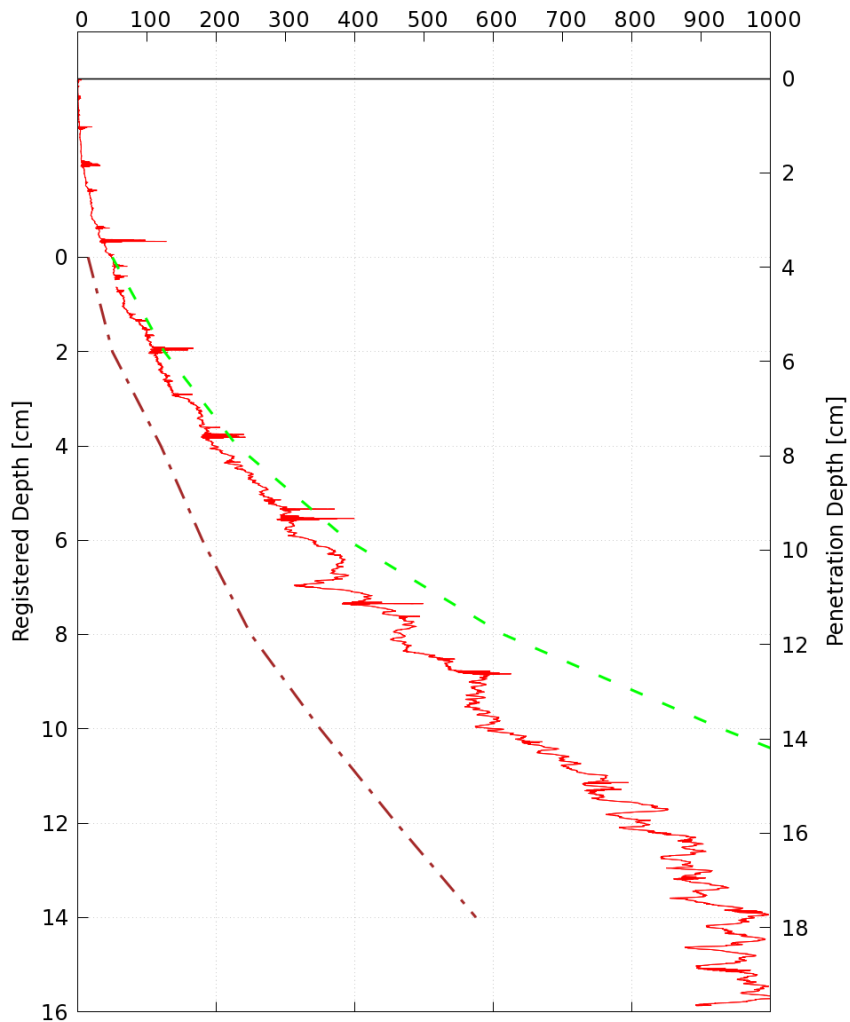
Shadow @ MSU HPCC,  
200 cores, ~3 hours per 0.05 s step.



# Cone penetrometer test large particle beds

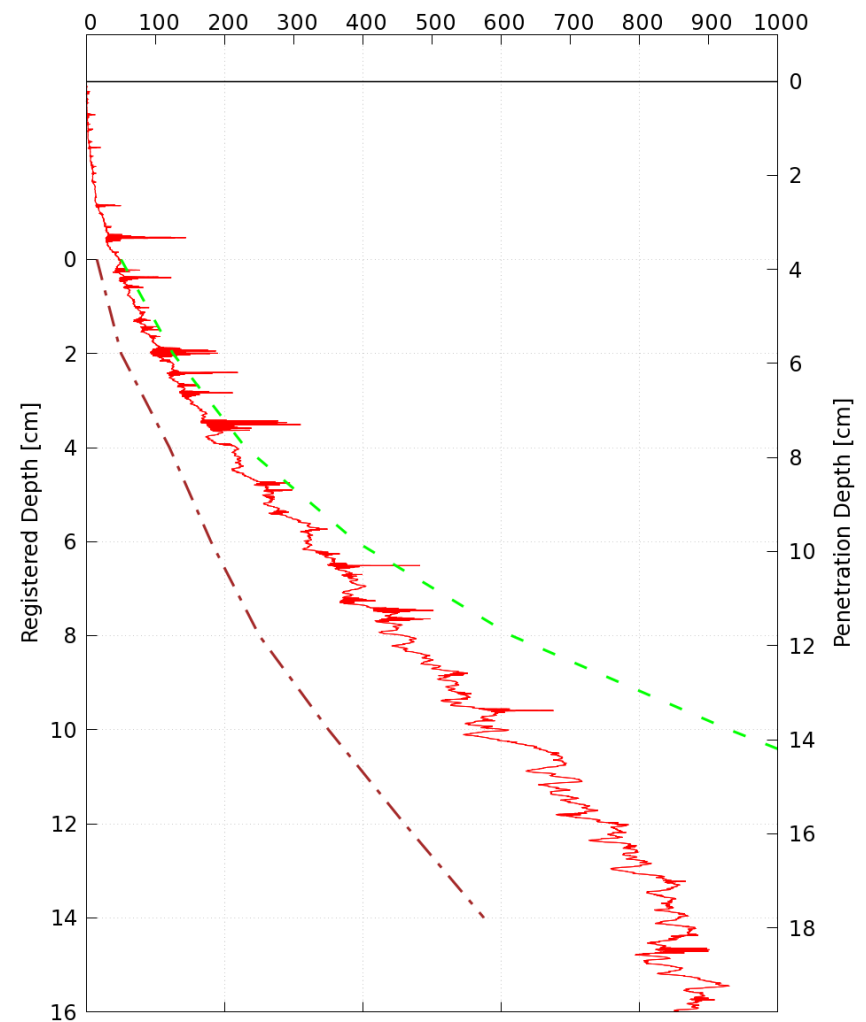
24 x 21 x 8.75 in<sup>3</sup>

Load [kN/m<sup>2</sup>]



48 x 21 x 8.75 in<sup>3</sup>

Load [kN/m<sup>2</sup>]





# Conclusions

Coupled DEM-LBM model of shear thickening in fluid-particle suspensions demonstrates capability to model fluid-soil interaction

Discrete Element Method can qualitatively predict pull-slip relationship for a wheel in granular media

Calibration and validation of DEM parameters is of utmost importance

ERDC HPCMP computational resources provide invaluable tool for large scale DEM/LBM studies

# Acknowledgments

DEM code origin: ERDC - John Peters, David Horner, Alex Carrillo

LBM code origin: MSU - Sergio Felicelli, Hebi Yin

LBM parallelization, HDF5 output and restart, DEM and LBM visualization: Bohumir Jelinek

DEM calibration: Clay Goodman

Experimental guide, DROVE database: George Mason, Farshid Vahedifard, Jody Priddy

LBM-DEM coupling: Bohumir Jelinek, Daniel Johnson