Loading Response of Densely Packed Particle assemblies in Fluid

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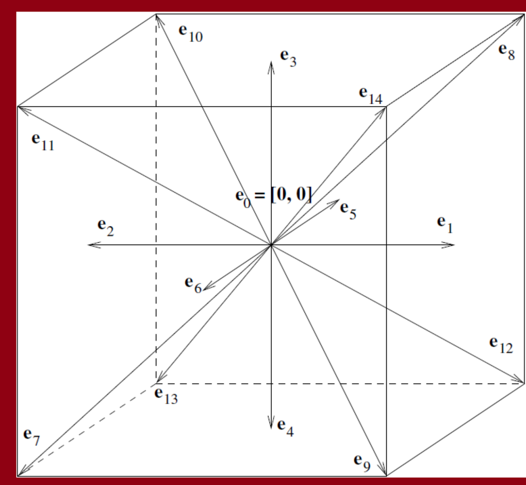
http://www.cavs.msstate.edu/projects/dem_lbm



Objective

Understanding mechanical properties of granular media in fluid, such as saturated soils, submerged road foundations, or flooded track ballasts, is of a great benefit in the design of transportation systems. The objective of this research is to couple the Discrete Element Method (DEM) and Lattice Boltzmann Method (LBM) in order to accurately simulate mechanical properties of closely packed particle assemblies in fluids.

Current State

- Implemented parallel LBM flow module
  - Calculated force and torque exerted by fluid on spherical particles
  - Validated LBM module through test cases
- Coupled LBM module with a DEM code
- Validated a coupled system with reference simulations including:
  - Drafting-Kissing-Tumbling
  - Large Scale Sedimentation Case
- Initial simulations of biaxial (2D) loading of un-drained granular media

LBM theory

LBM density distribution functions

The macroscopic fluid density ρ at each lattice point is a sum of the distribution functions at that lattice point:

$$\rho = \sum_{i=0}^{14} f_i \tag{1}$$

Fluid velocity is a weighted sum of lattice velocities:

$$\boldsymbol{u} = \frac{\sum_{i=0}^{14} f_i \boldsymbol{e}_i}{\sum_{i=0}^{14} f_i} = \frac{\sum_{i=0}^{14} f_i \boldsymbol{e}_i}{\rho} \tag{2}$$

Equilibrium

For the D3Q15 lattice, the equilibrium distribution function f_i^{eq}, given the macroscopic velocity $\boldsymbol{u}(\boldsymbol{r})$ and density $\rho(\boldsymbol{r})$, is:

$$f_i^{\mathrm{eq}}(\boldsymbol{r}) = w_i \rho(\boldsymbol{r}) \left(1 + 3\frac{\boldsymbol{e}_i \cdot \boldsymbol{u}(\boldsymbol{r})}{c^2} + \frac{9}{2}\frac{(\boldsymbol{e}_i \cdot \boldsymbol{u}(\boldsymbol{r}))^2}{c^4} - \frac{3}{2}\frac{\boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{u}(\boldsymbol{r})}{c^2}\right), \tag{5}$$

with the lattice velocity $c = \Delta x / \Delta t$ and the weights

$$w_i = \begin{cases} 4/9 & i = 0 \\ 1/9 & i = 1,2,3,4 \\ 1/36 & i = 5,6,7,8. \end{cases} \tag{6}$$

Time evolution of the distribution functions

Using the collision model of Bhatnagar-Gross-Krook (BGK) with a single relaxation time, evolution of the distribution functions is

$$f_i(\boldsymbol{r} + \boldsymbol{e}_i \Delta t, t + \Delta t) = f_i(\boldsymbol{r}, t) + \frac{1}{\tau_{\mathrm{u}}}\left(f_i^{\mathrm{eq}}(\boldsymbol{r}, t) - f_i(\boldsymbol{r}, t)\right), i = 0 \ldots 8 \tag{3}$$

Δt ... time step

$\boldsymbol{r}$ ... space position of a lattice site

t ... time position of a lattice site

τ_{u} ... relaxation parameter for the fluid flow.

Viscosity

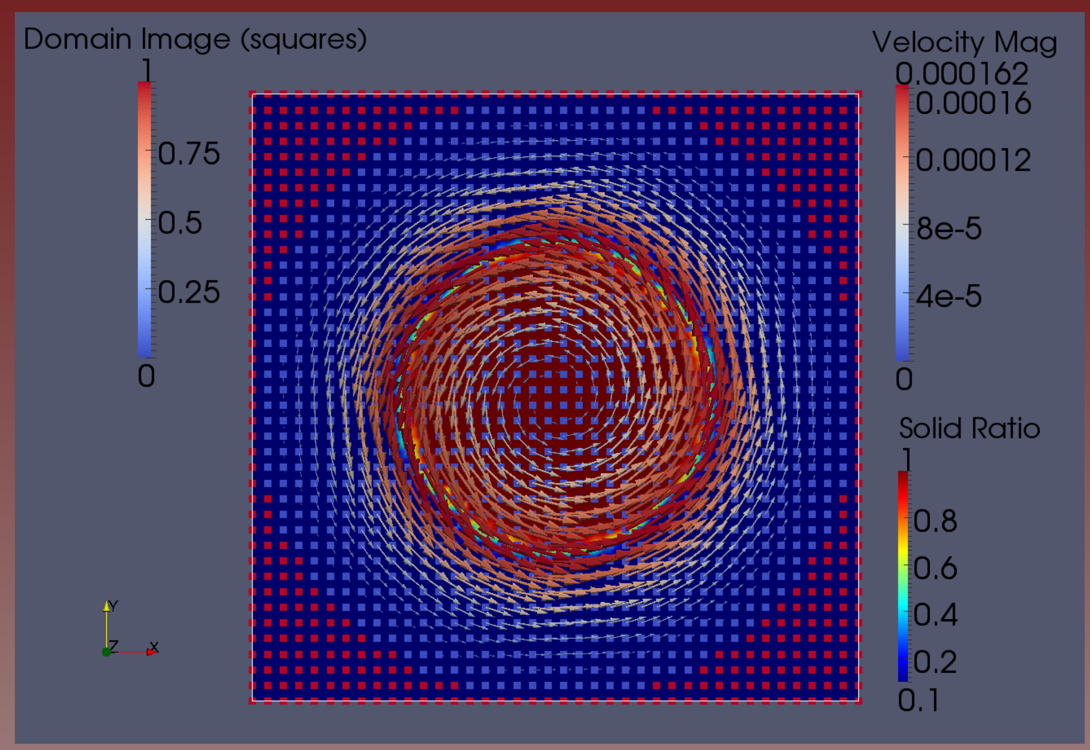
The relaxation parameter τ_{u} specifies how fast each particle distribution function f_i approaches its equilibrium f_i^{eq}. Kinematic viscosity ν is related to the relaxation parameter τ_{u}, the lattice spacing Δx, and the simulation time step Δt by

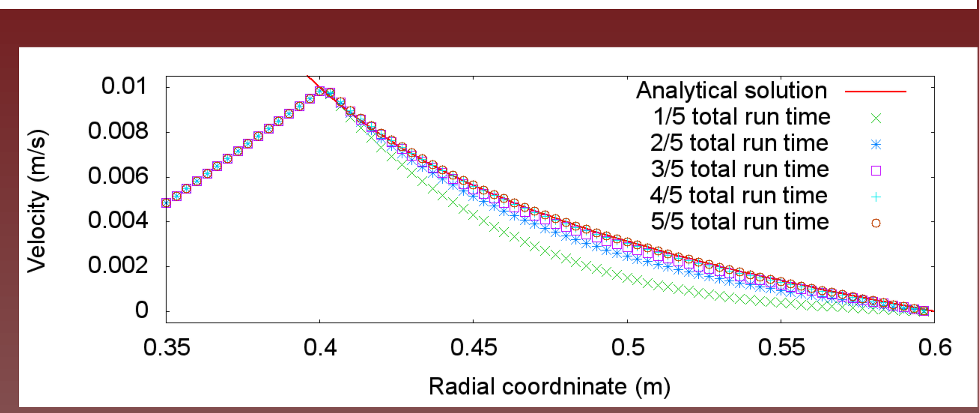
$$\nu = \frac{\tau_{\mathrm{u}} - 0.5}{3}\frac{\Delta x^2}{\Delta t}. \tag{4}$$

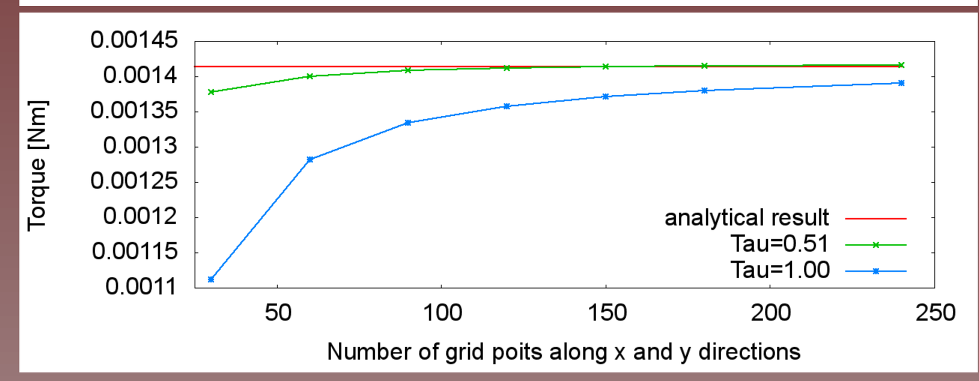
Depending on the dimensionality d of the modeling space and a chosen set of the discrete velocities $\boldsymbol{e}_i$, the corresponding equilibrium particle distribution function can be found.

Results

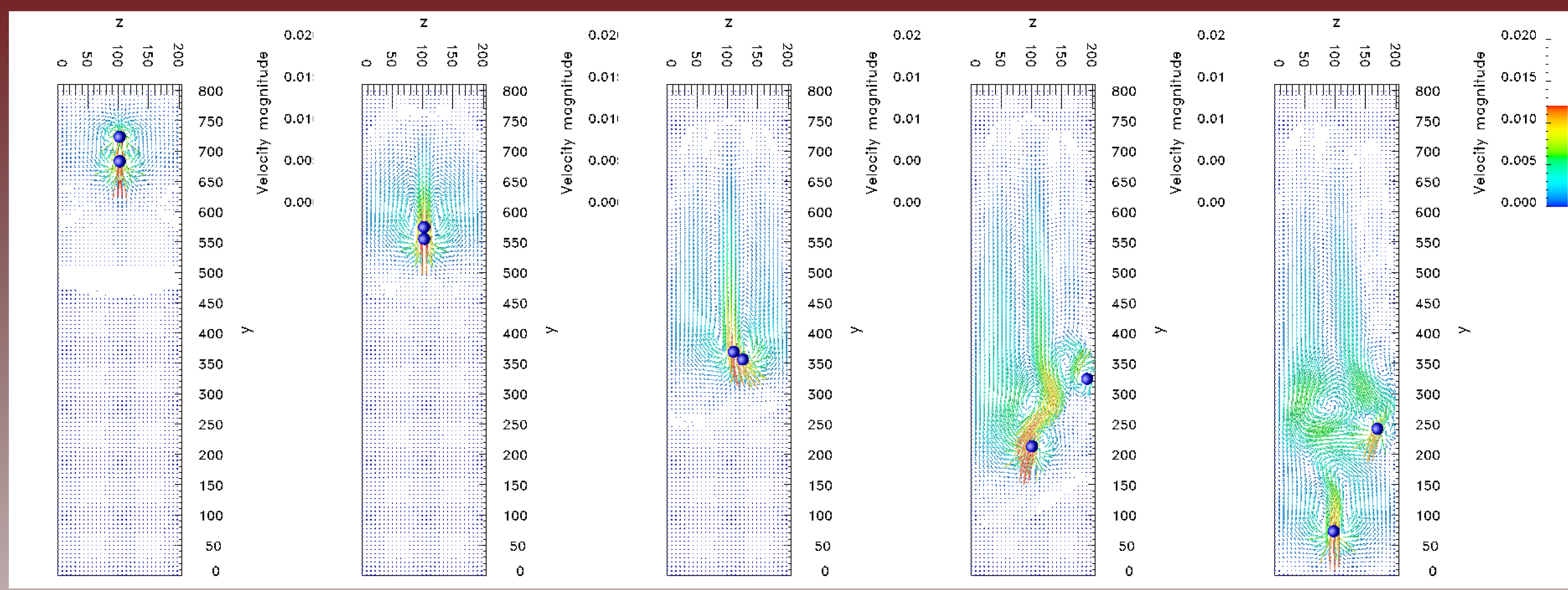
Accuracy - Torque calculation



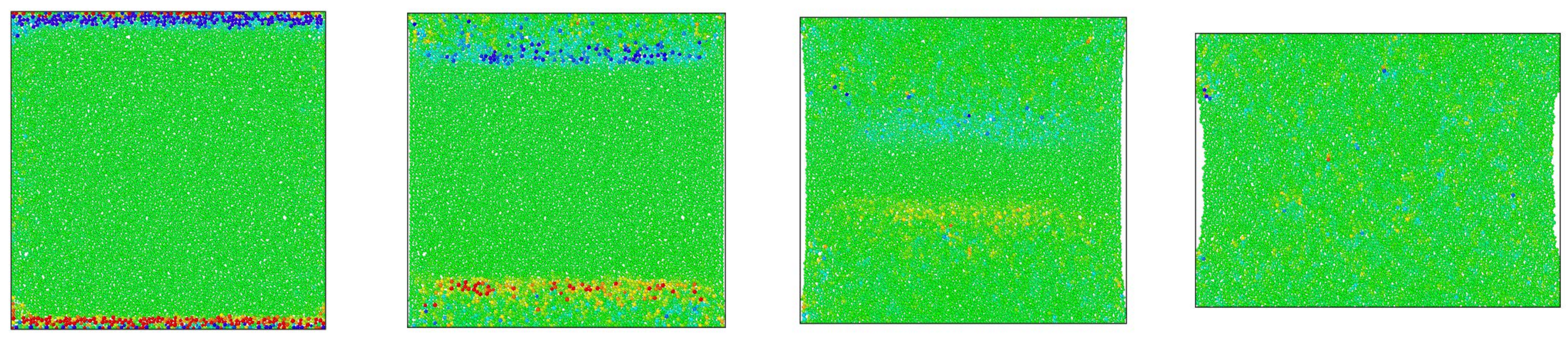




DKT – two particles under gravity in fluid



Drained biaxial test



Color represents vertical component of force on particles upon compaction

Sedimentation